Première journée nationale SHM-France

Accuracy of flaw localization algorithms: application to structures monitoring using ultrasonic guided waves

Alain Le Duff

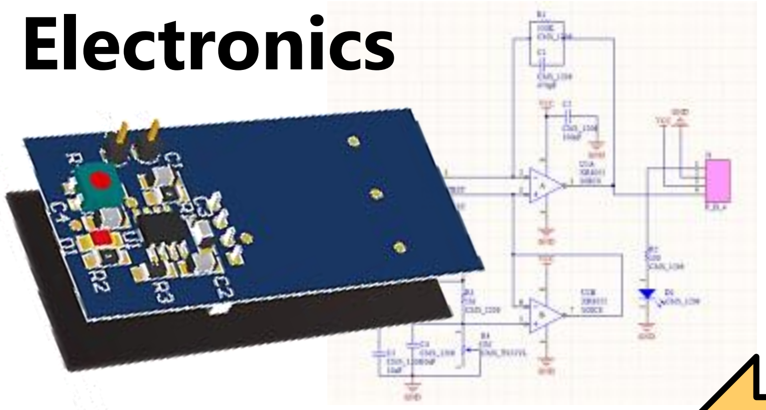
Groupe Signal Image & Instrumentation (GSII), Groupe ESEO, Angers, France
Laboratoire d'Acoustique de l'Université du Maine (LAUM UMR CNRS 6613), Le Mans, France

Who am I?

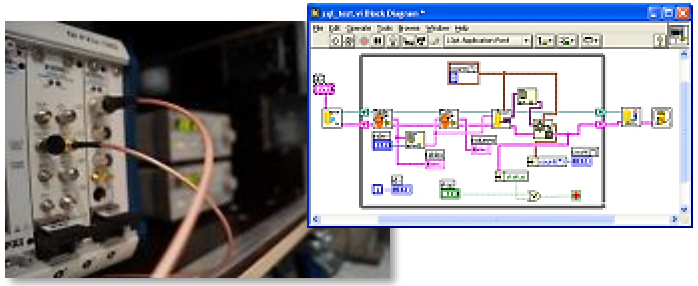
- **Alain Le Duff, Ing., PhD, HDR**
- **Teacher-researcher at ESEO, Angers, France**
  - Head of the Department of Electronics and Control Engineering
  - Member of the « Signal, Image & Instrumentation Group » (GSII)
- **Research fellow at LAUM, Le Mans, France**
  - Instrumentation and signal processing for acoustics
  - Field of applications : LDV, Musical acoustics, ENDT, SHM

NDT & SHM: Work context

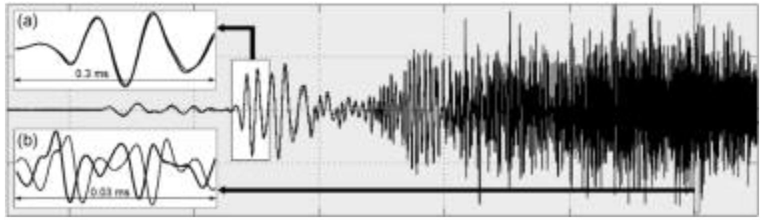
Electronics



Instrumentation



Signal processing



solutions

NDT & SHM

Materials

Aluminum

Composite

Concrete

Bio.

Methods

Active

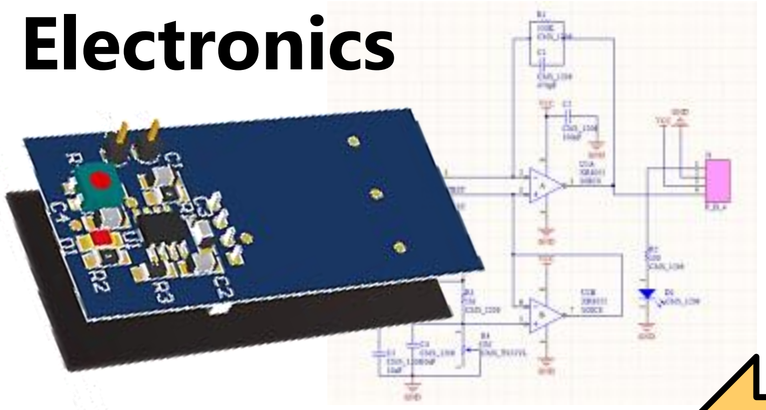
Guided waves, imaging
Coda Wave Interferometry (CWI)

Passive

Acoustic emission

NDT & SHM: Work context

Electronics



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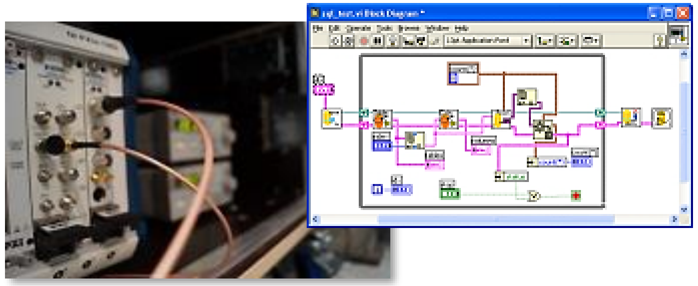

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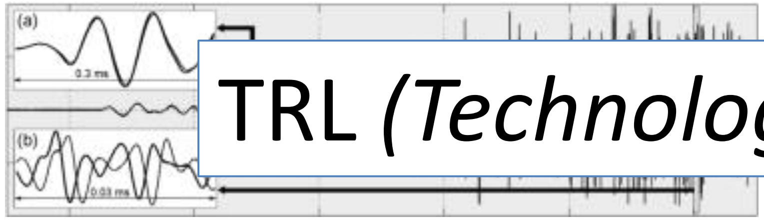

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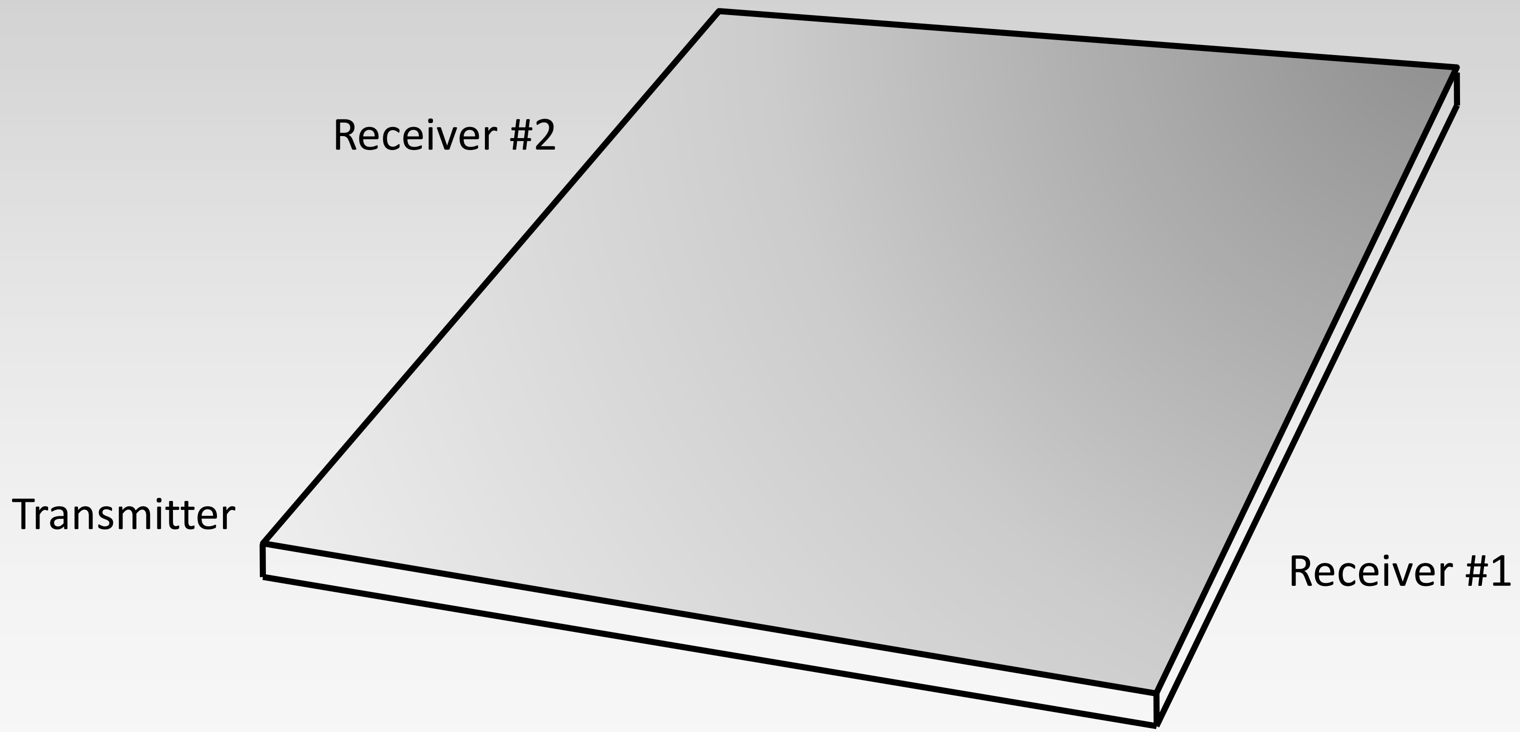
TRL *(Technology Readiness Level)* : 2 ⟷ 6

SHM Work context

- Guided waves (Lamb modes) in "plate" type structures;
- Use of active ultrasound method;
- Flaw localization in plate structures.



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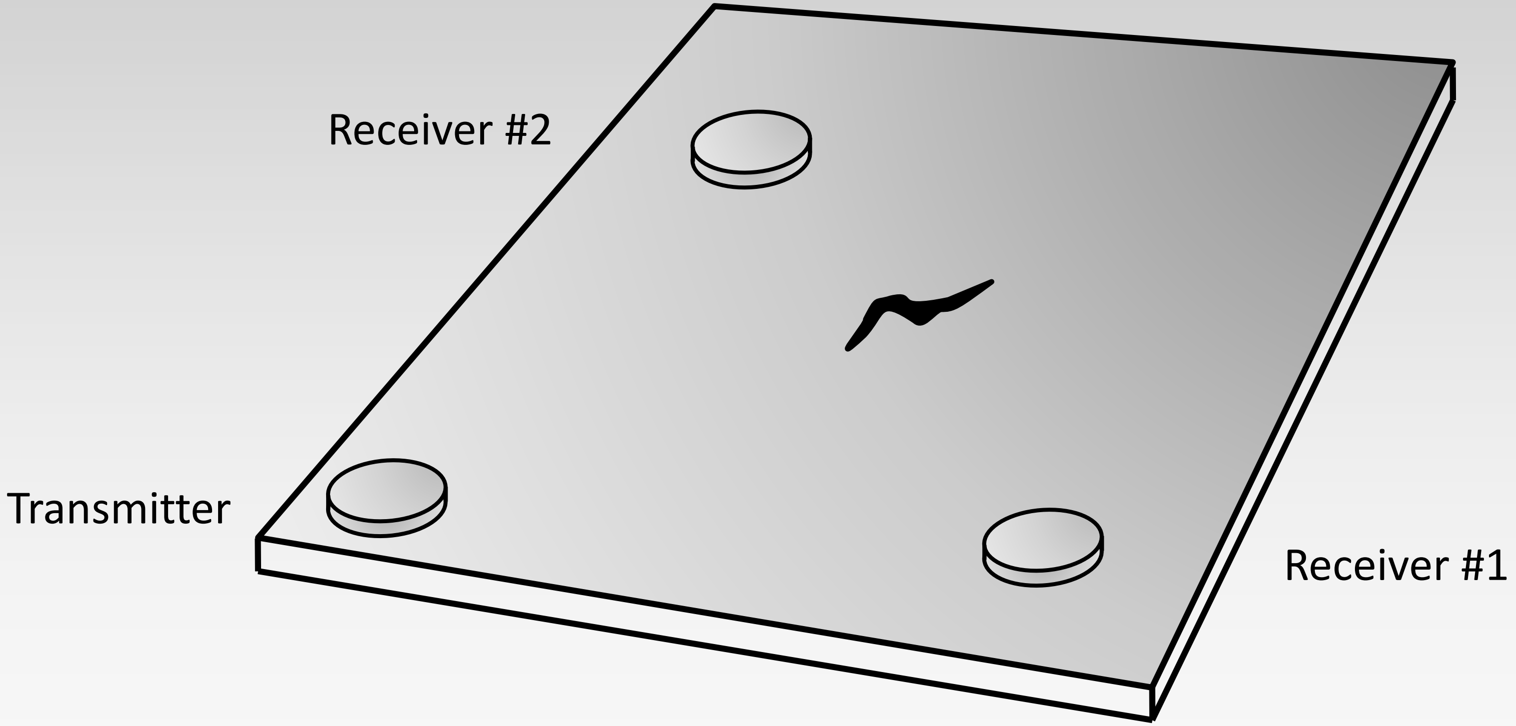


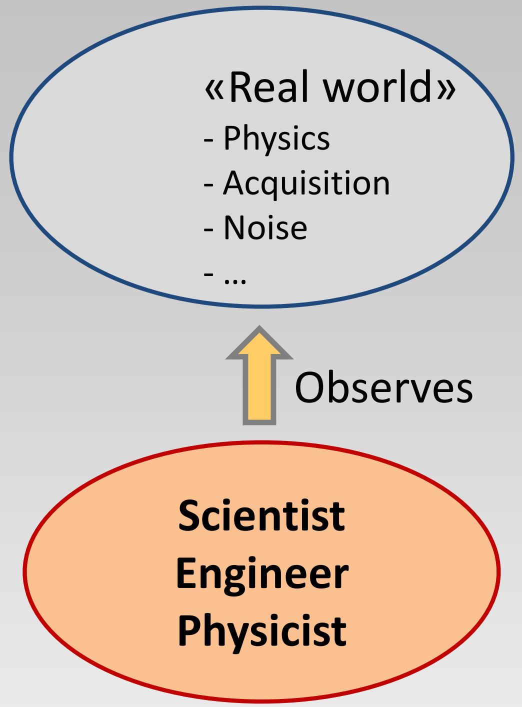
Methodological approach #2

«Real world»

- Physics
- Acquisition
- Noise
- …

Scientist
Engineer
Physicist

Methodological approach #2



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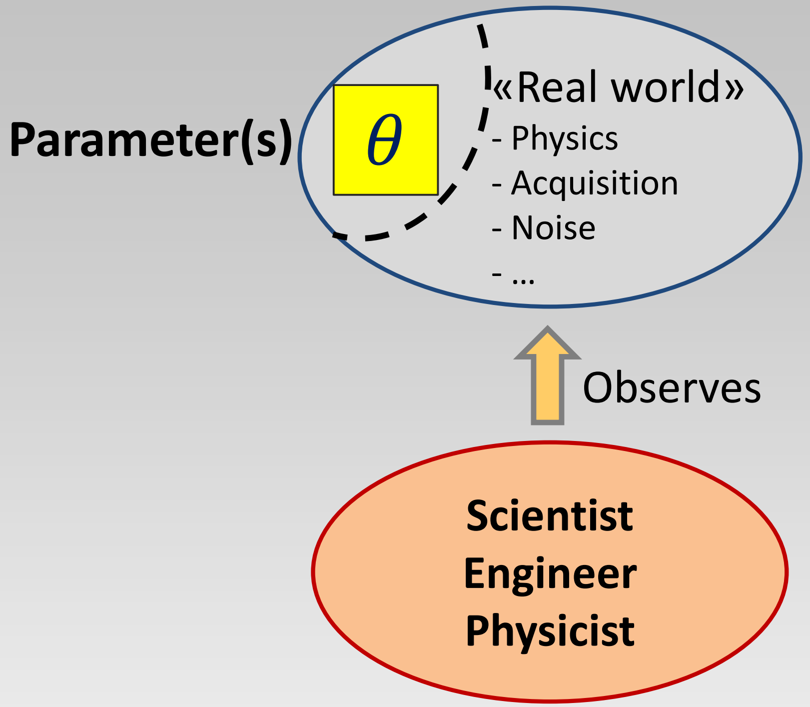

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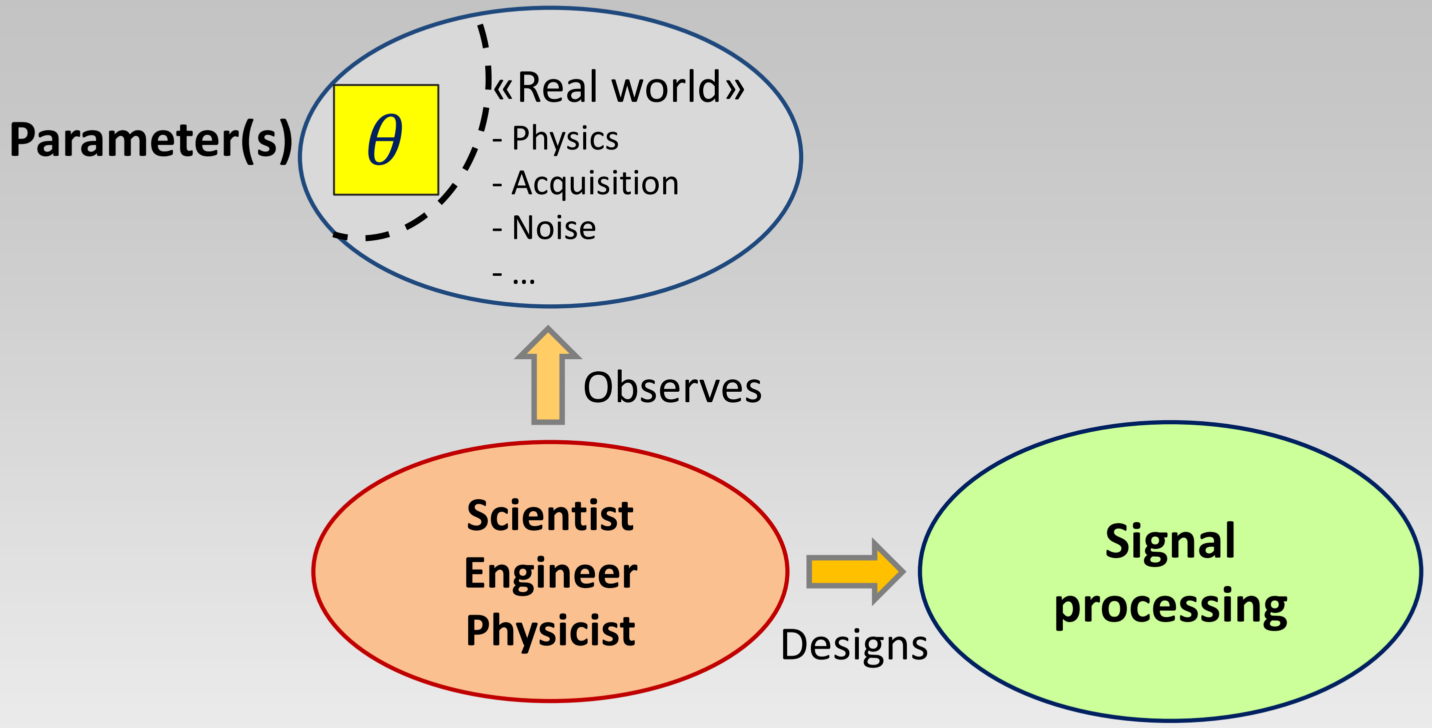

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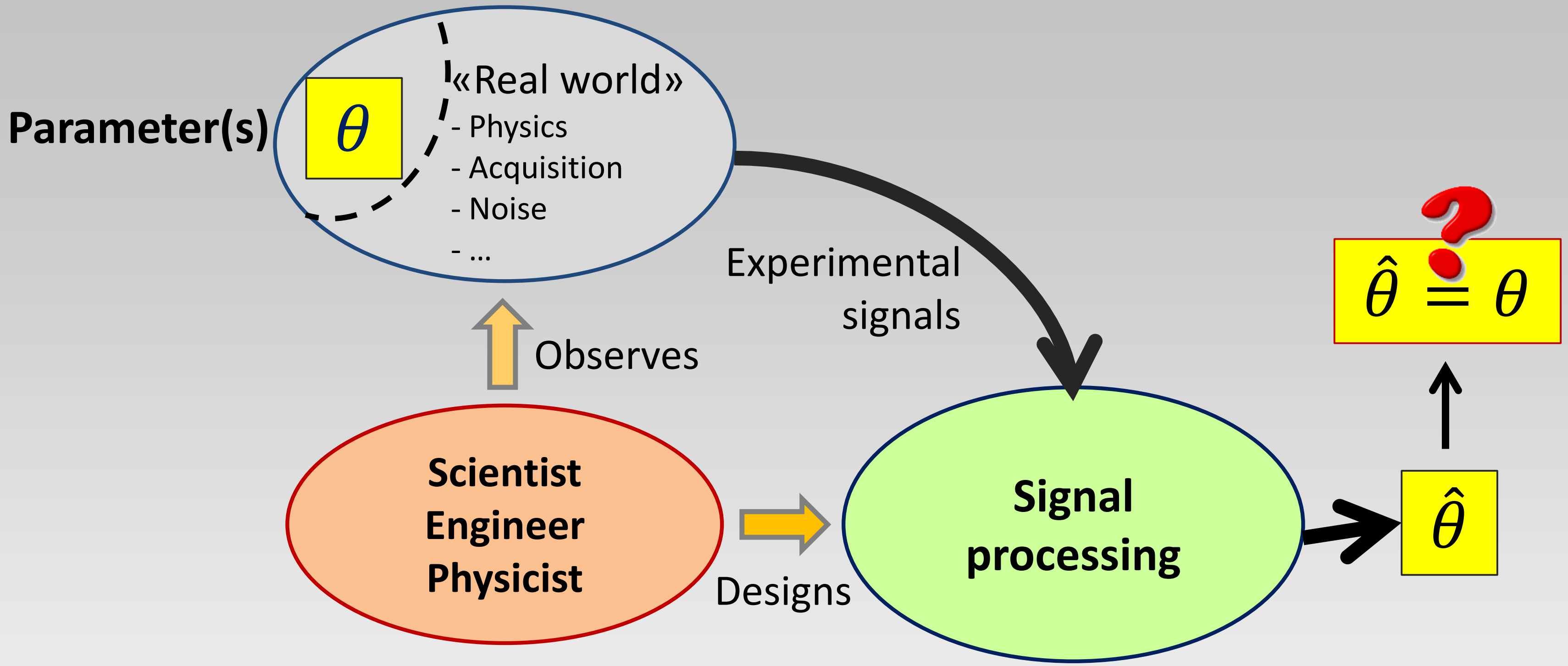

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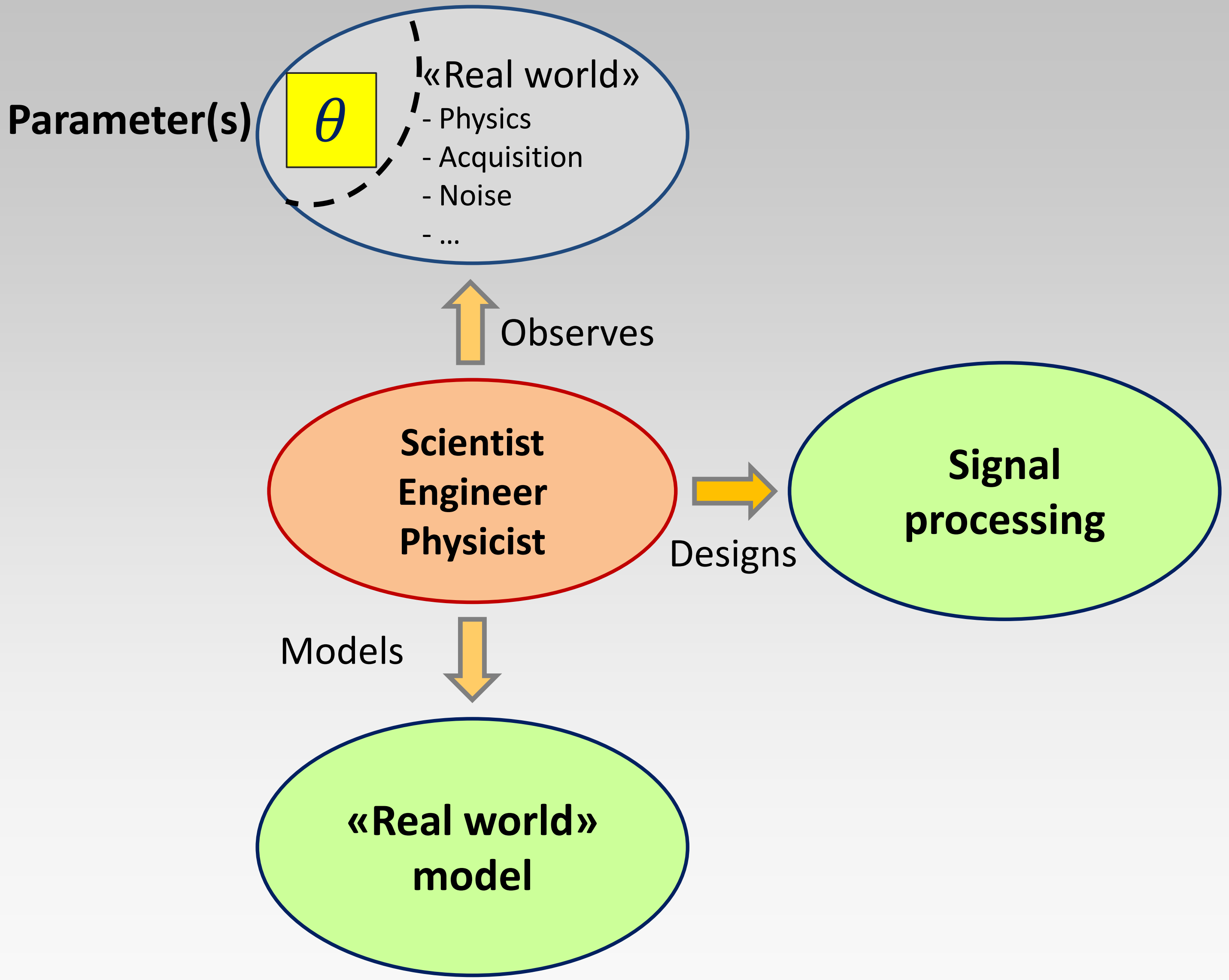

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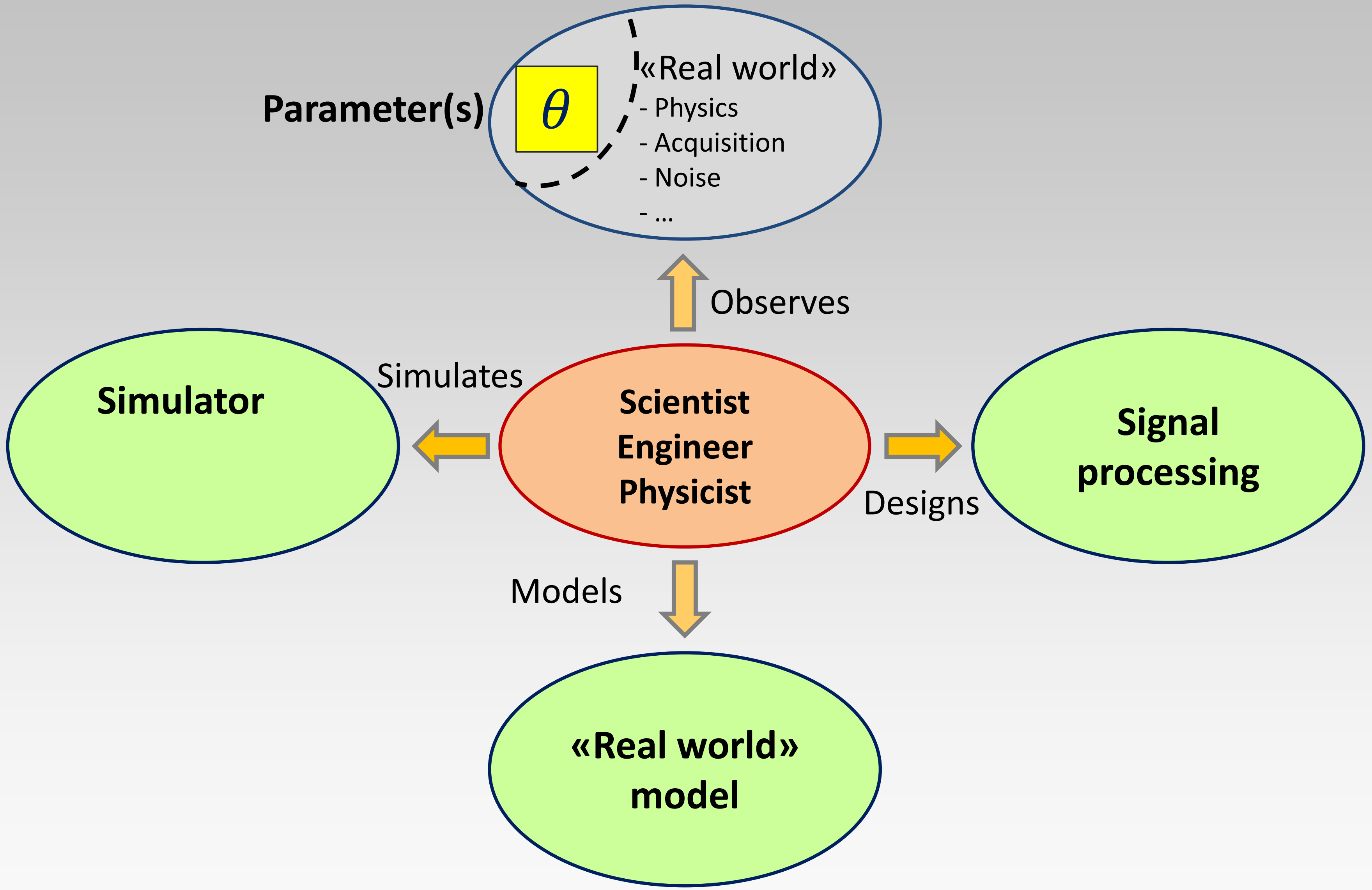

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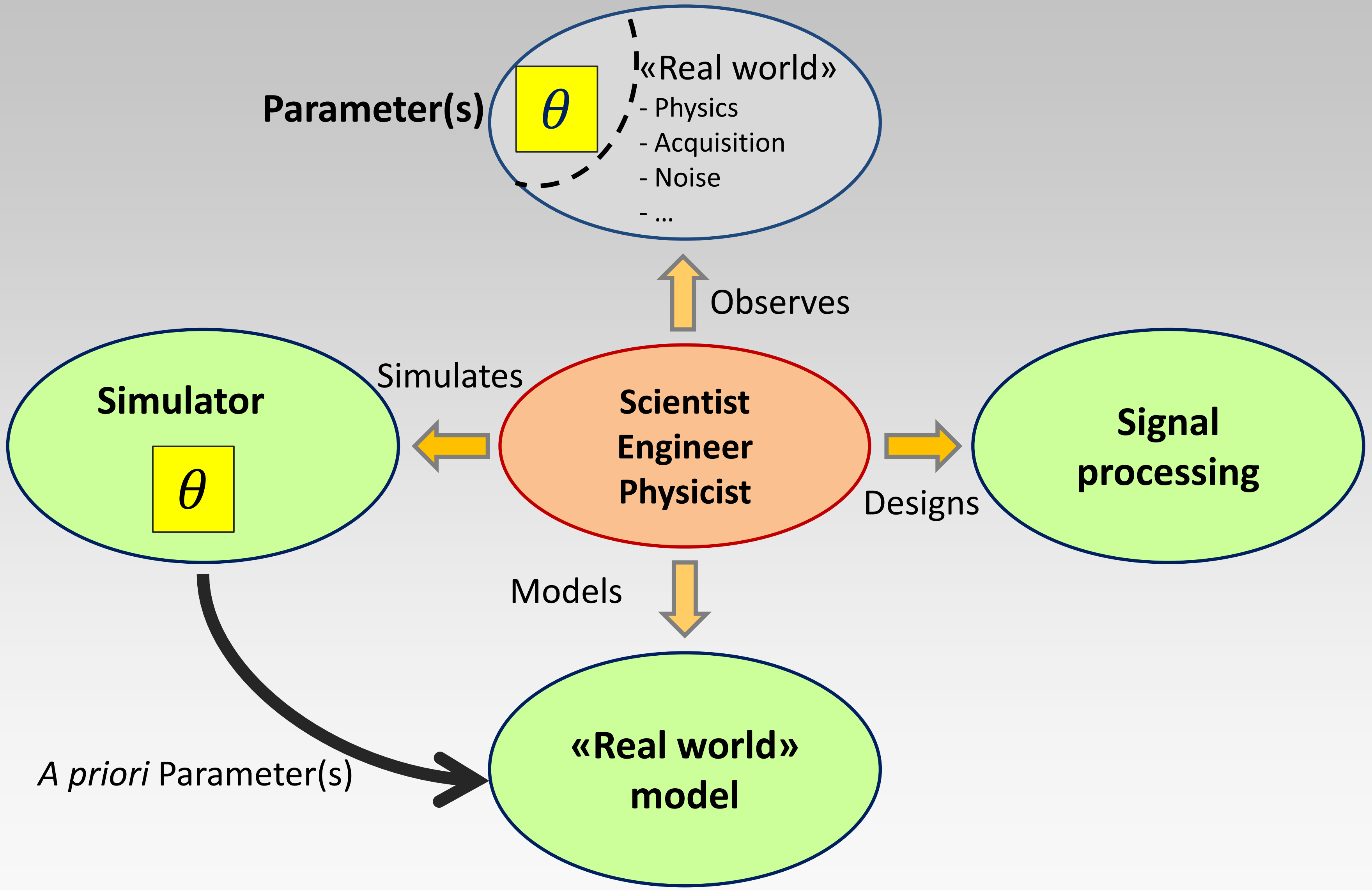

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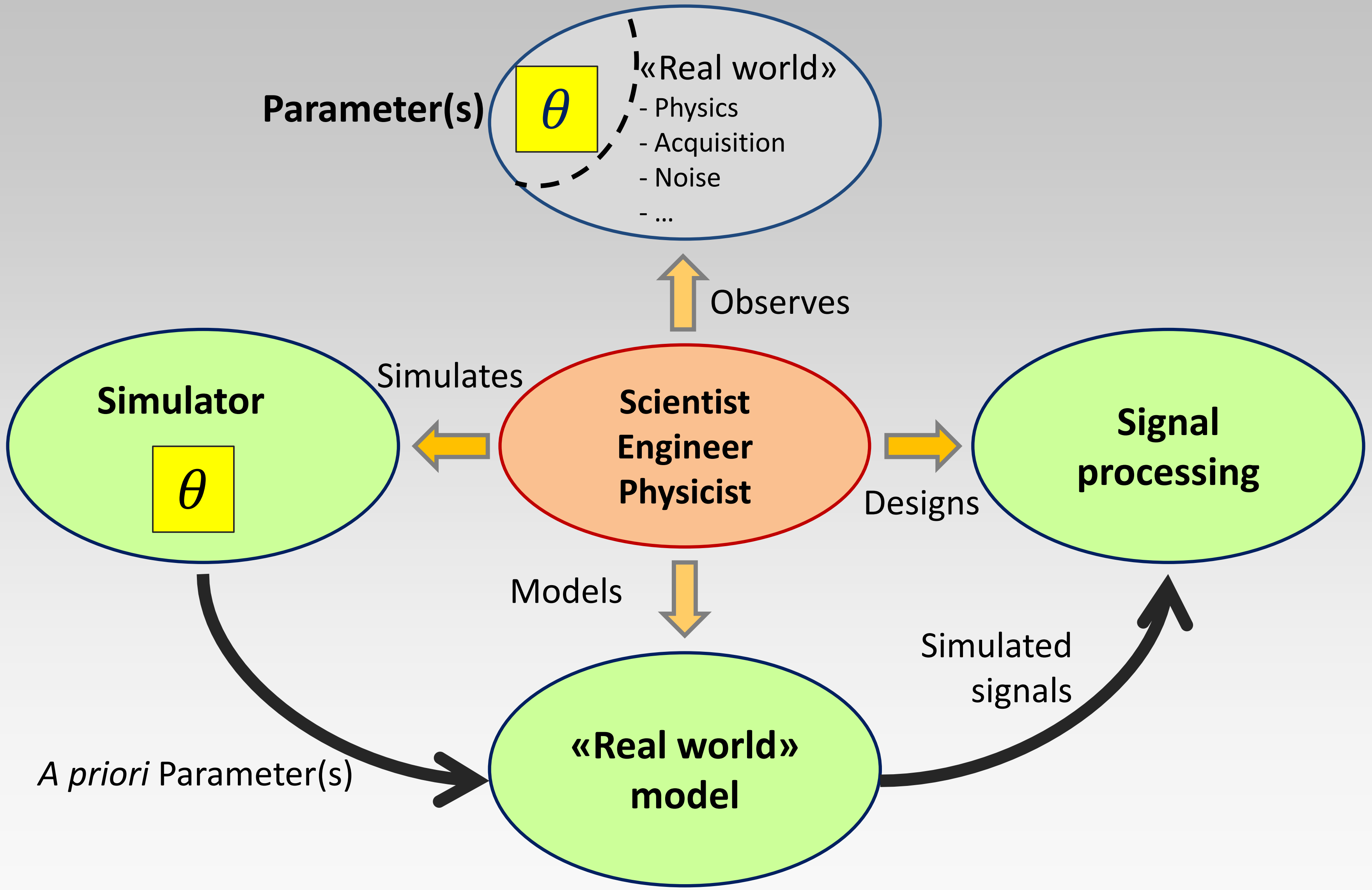

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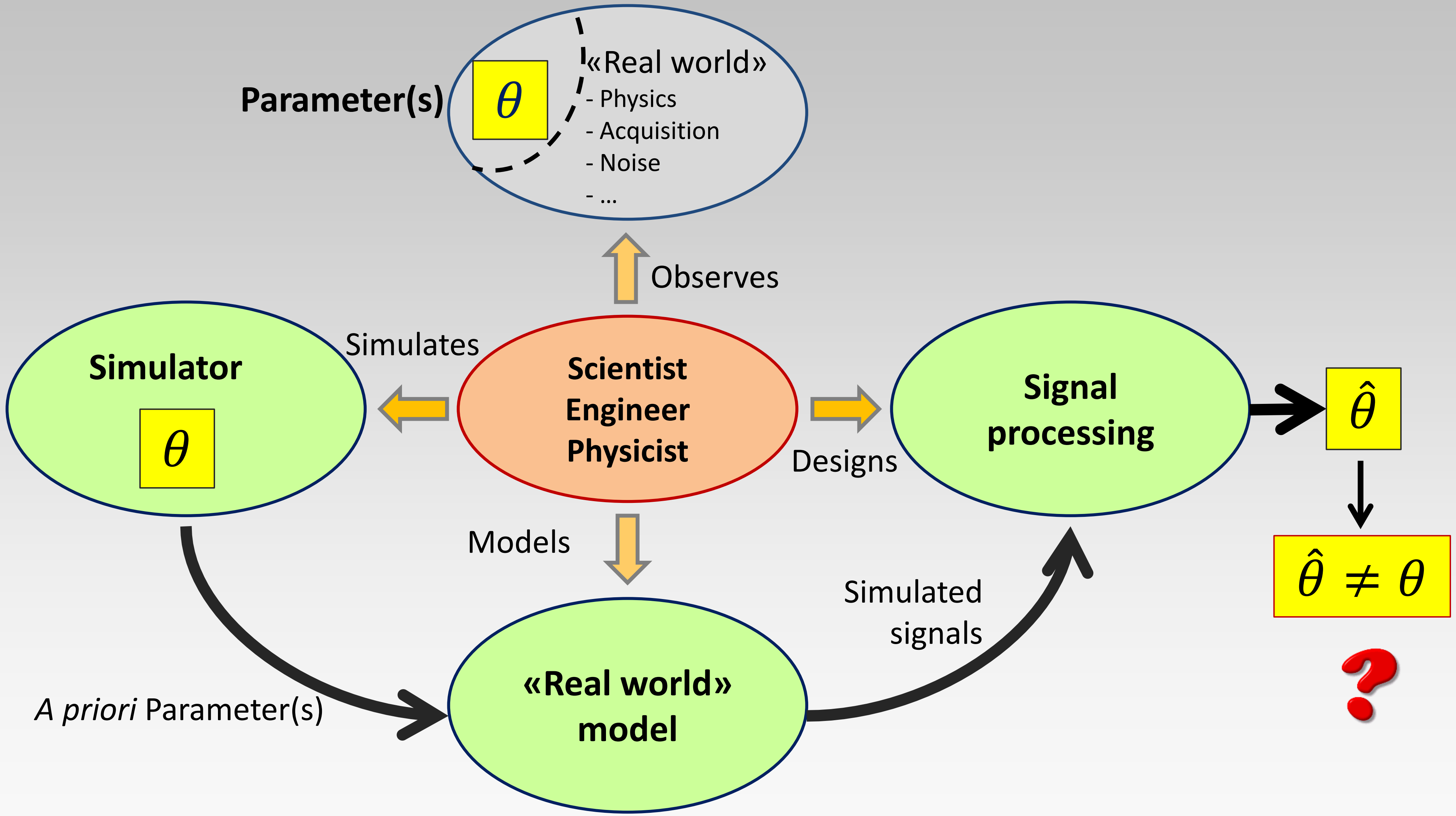

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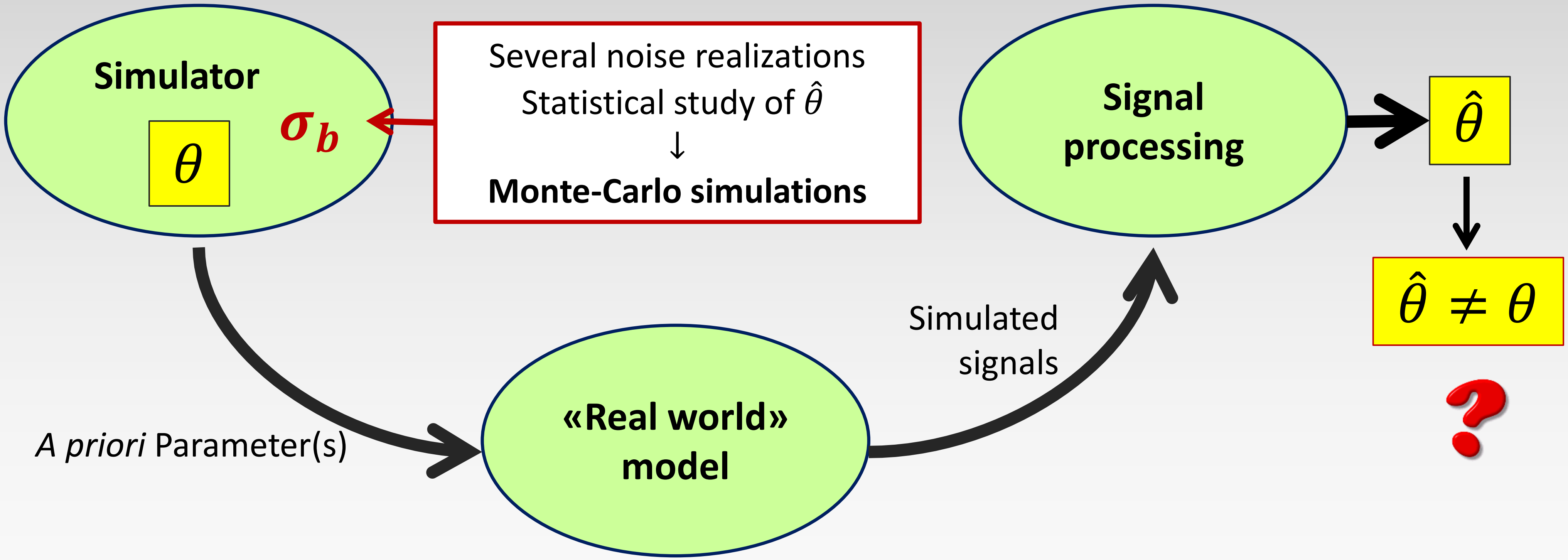

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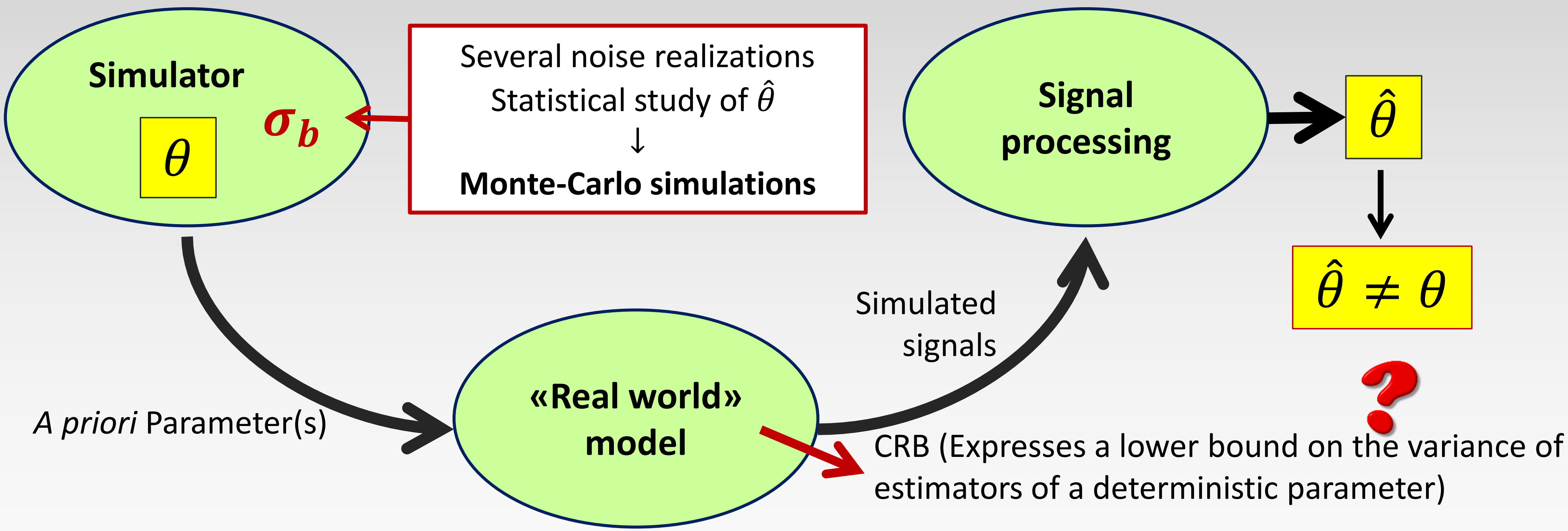

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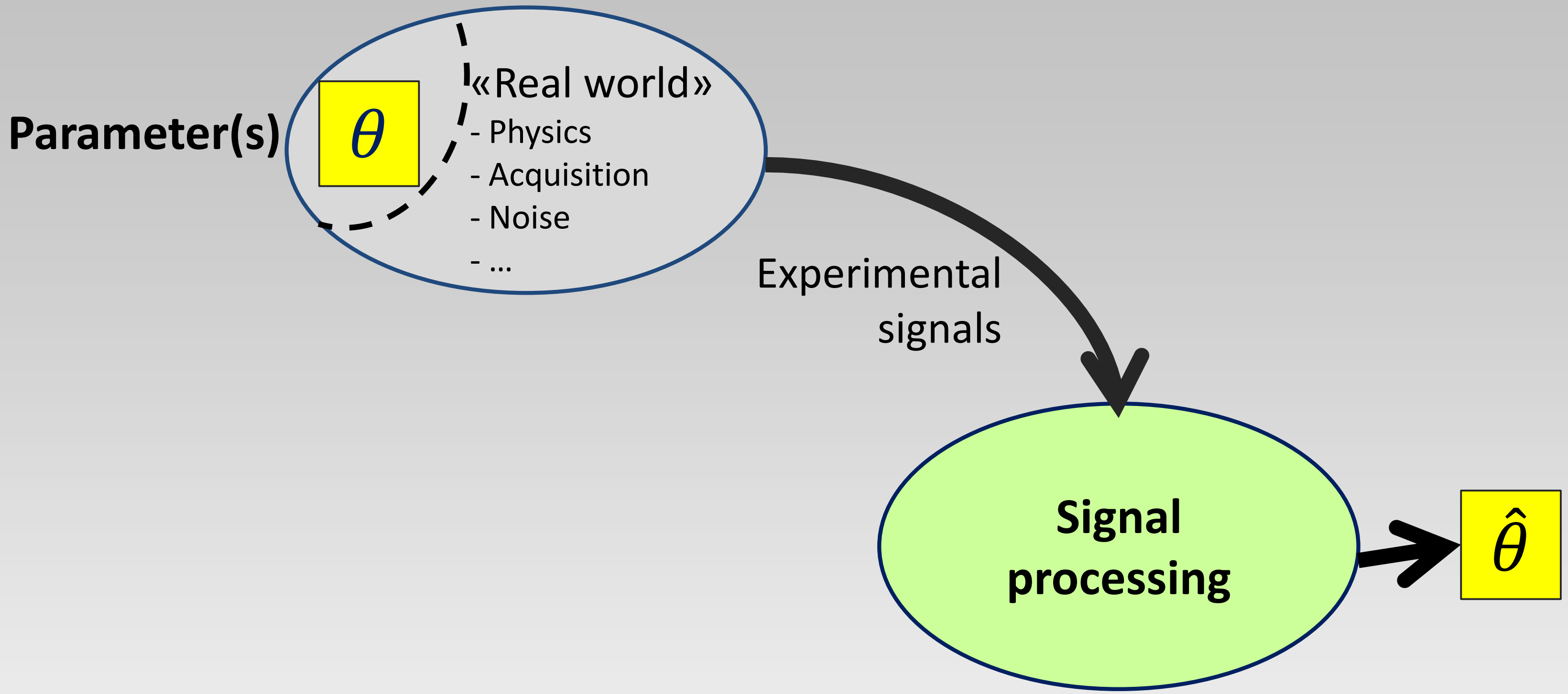

Methodological approach #3

- **2 examples**

1. Damage localization estimation

1. Temperature estimation

Methodological approach #3

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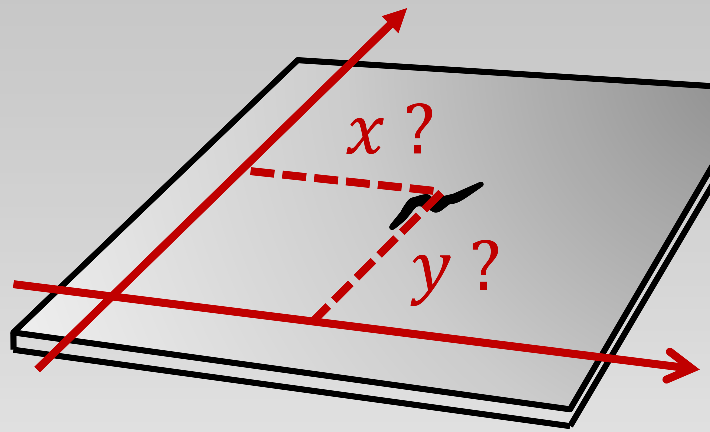

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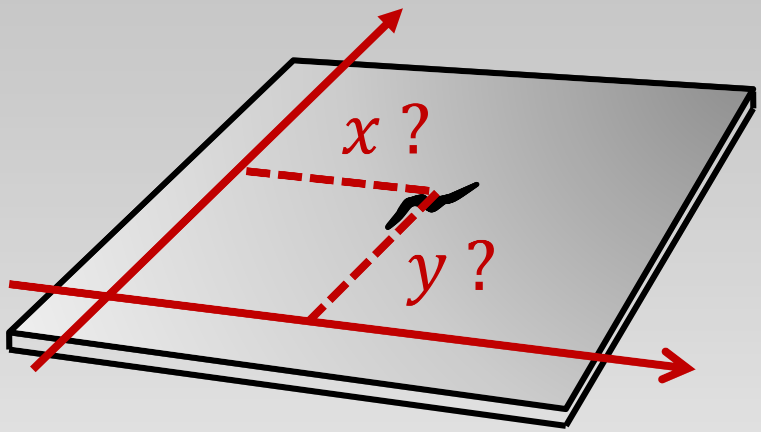
Methodological approach #3

- **2 examples**

1. Damage localization estimation



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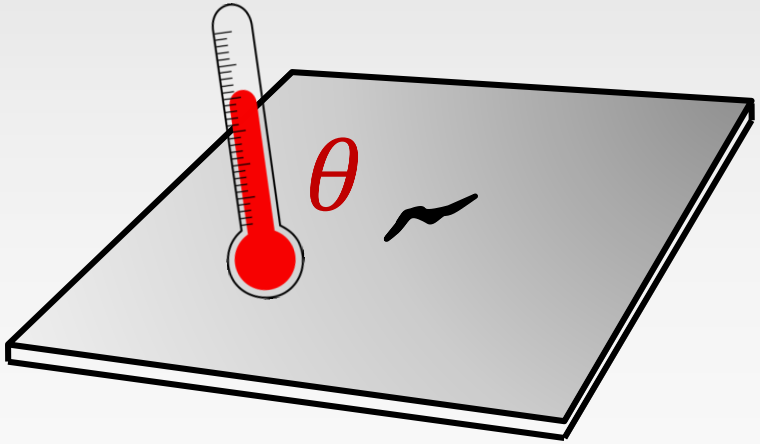
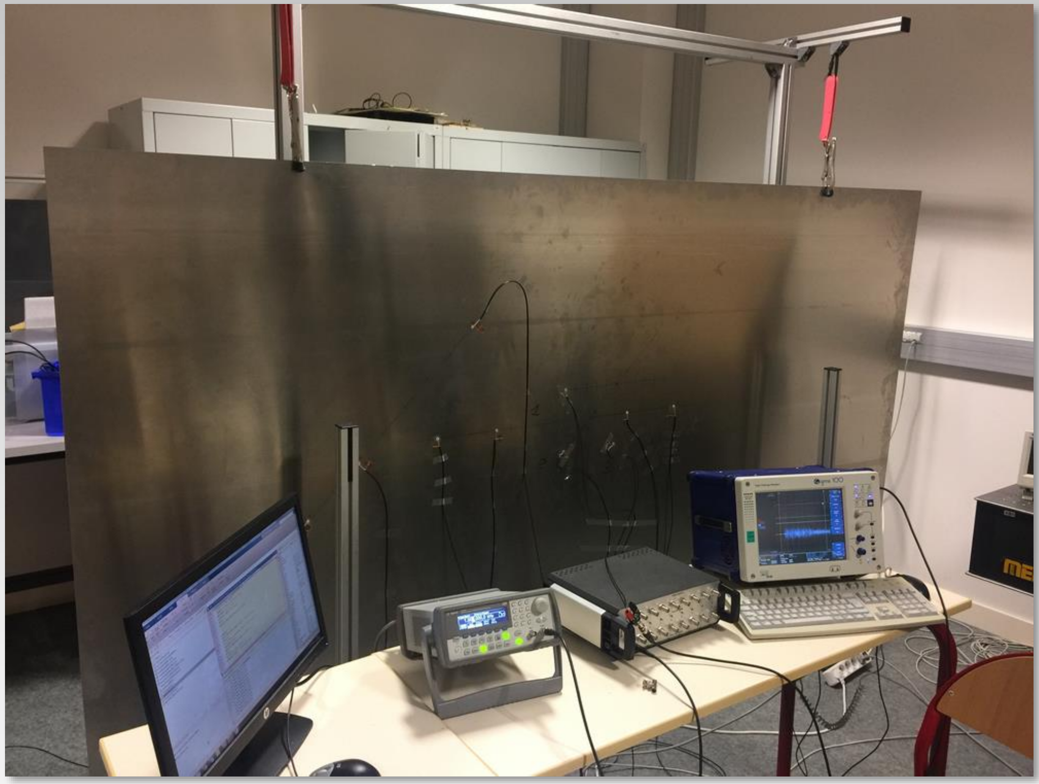


Example #1: damage localization in a plate

- **Context**
  - Array of piezoceramic sensors and actuators
  - TOF measurements
- **Localization accuracy depends on**
  - Spatial distribution of the transducers
  - Signal to Noise Ratio (SNR) of acquired signals
  - Defect location itself
- **Proposition**:
  - Study of the of damage localization → CRB
  - Experimental assesement of CRB in an aluminium plate using Guided Waves (GW)



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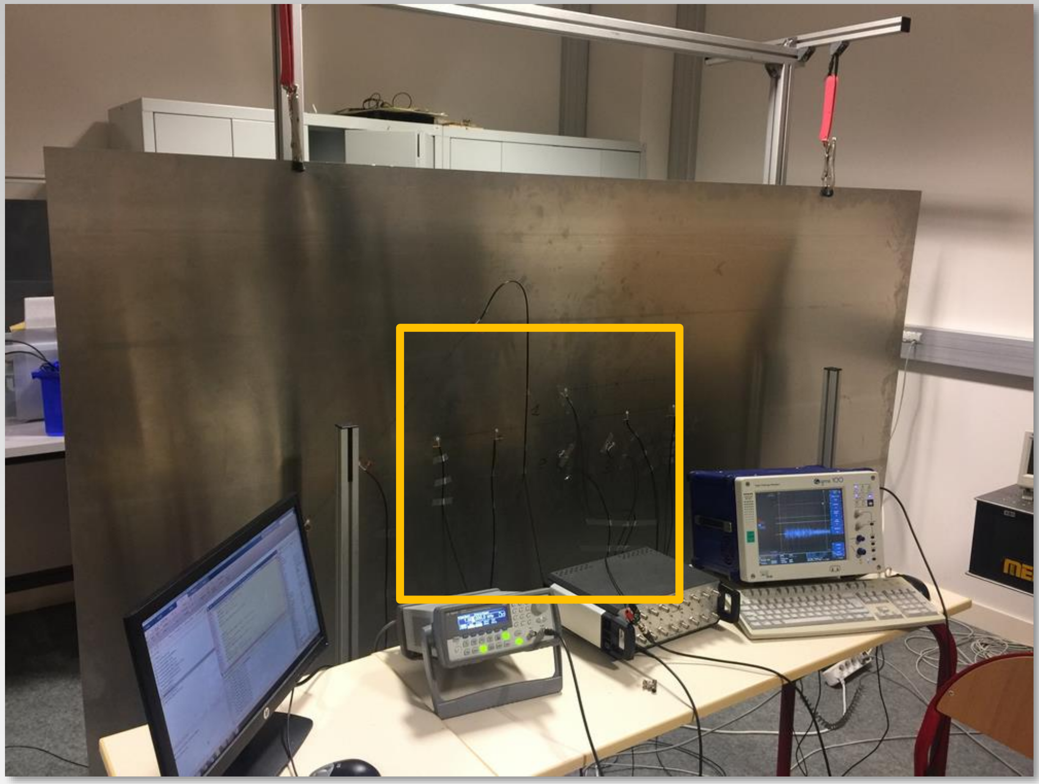

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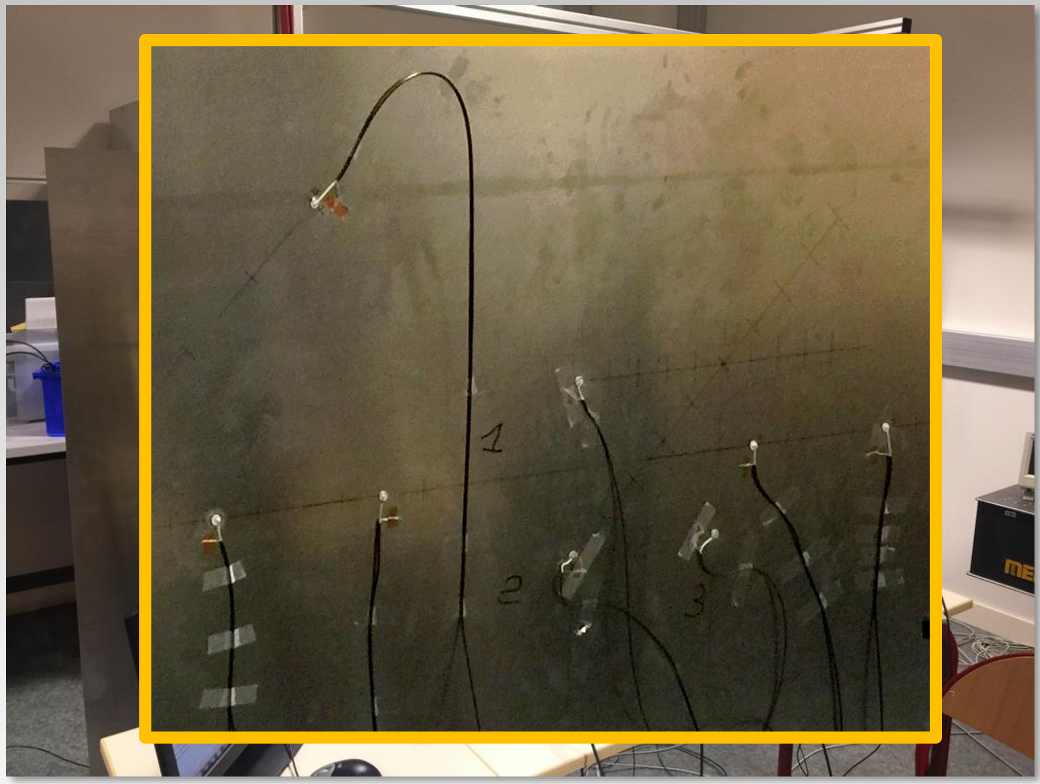
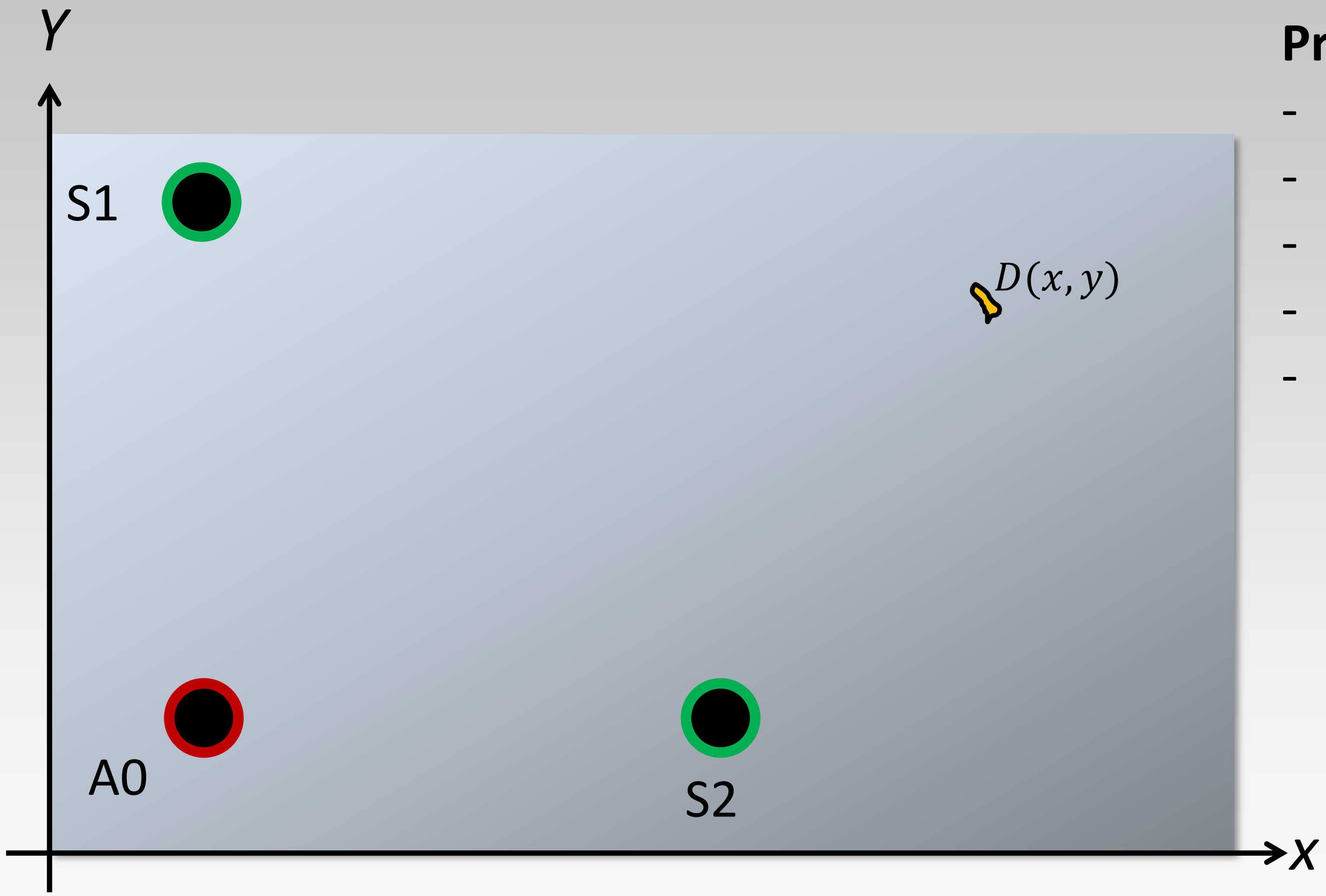


Principle of localization estimation #1

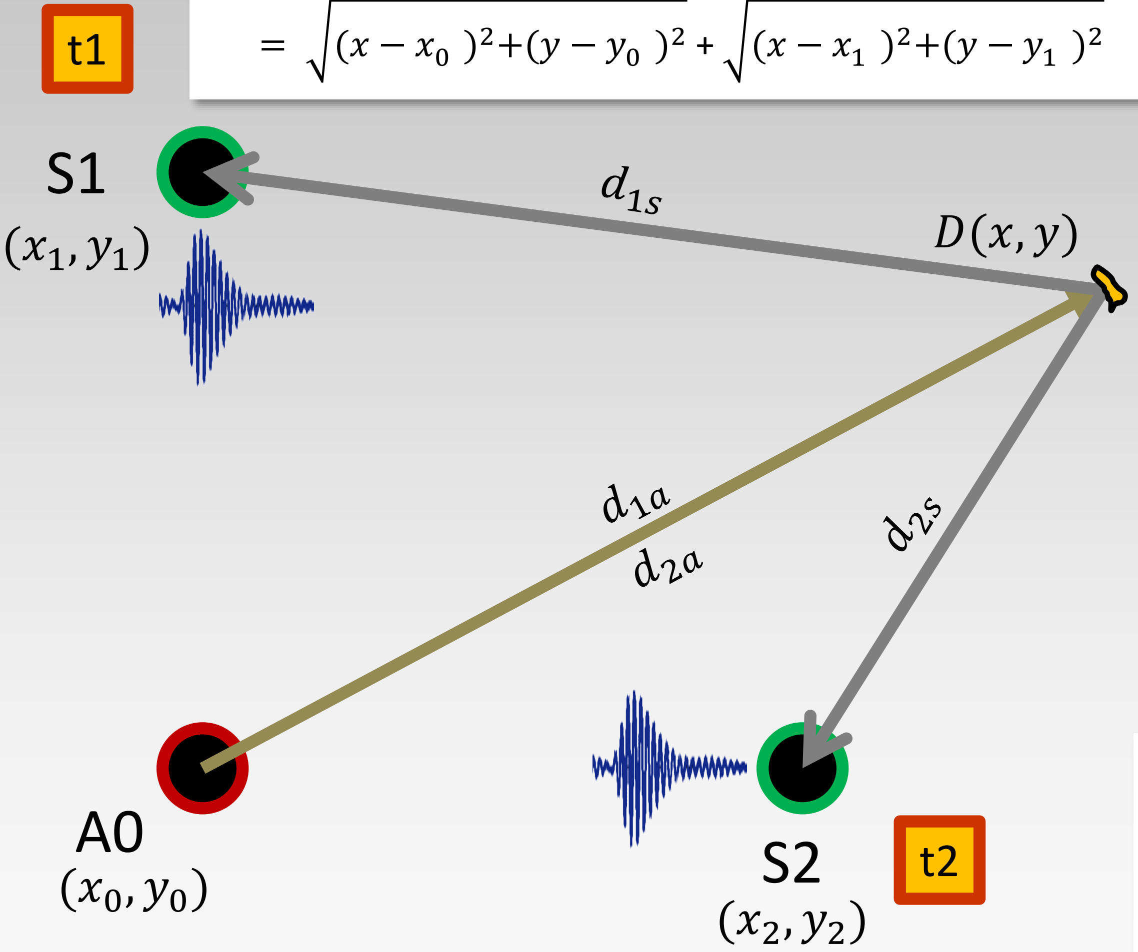


Problem:
- damage localization
- Isotropic plate
- guided waves
- TOF
- Array of 3 transducers distributed on the struture.

Principle of localization estimation #2

Distance between A0 and S1

$$d_1 = d_{1a} + d_{1s} = c \cdot t_1$$
$$= \sqrt{(x - x_0)^2 + (y - y_0)^2} + \sqrt{(x - x_1)^2 + (y - y_1)^2}$$



Distance between A0 and S2

$$d_2 = d_{2a} + d_{2s} = c \cdot t_2$$
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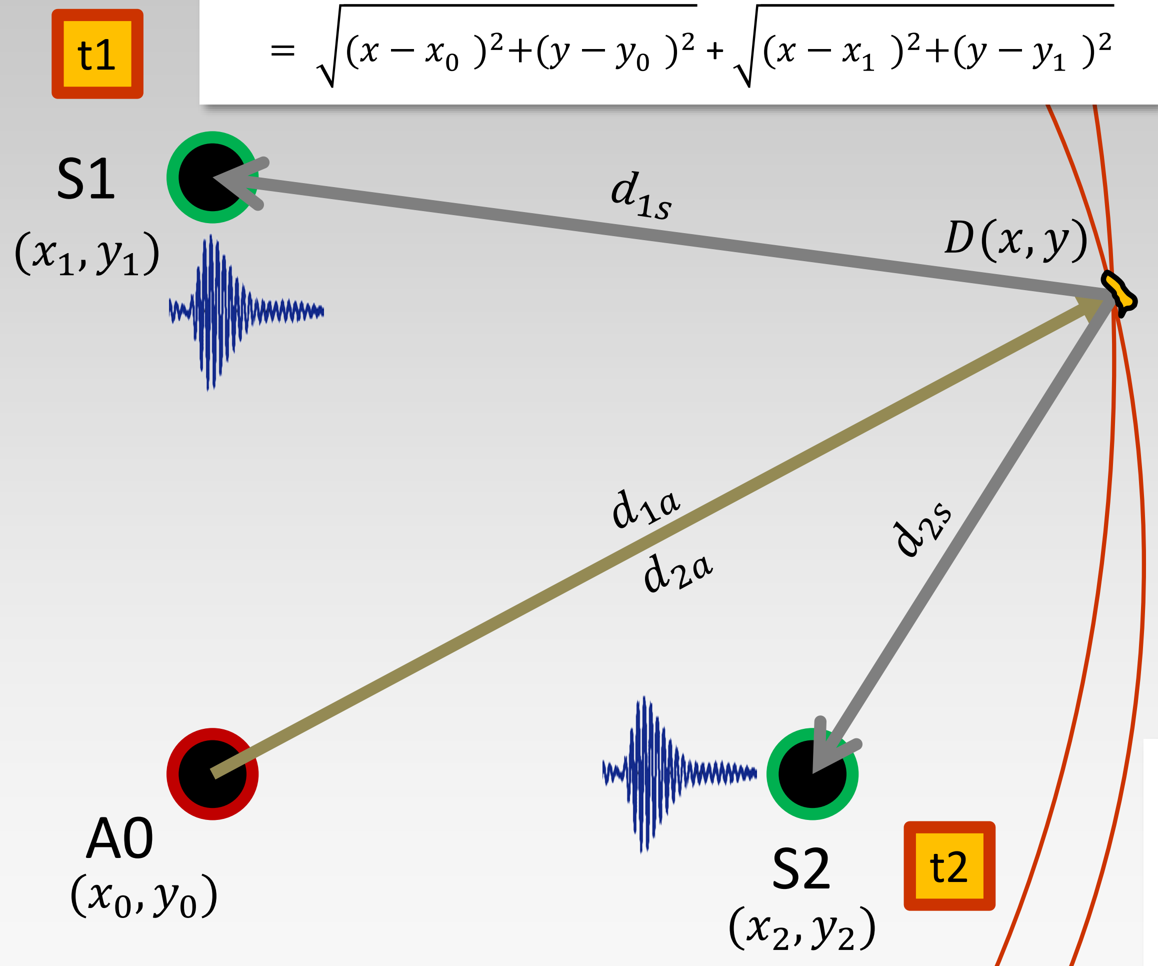

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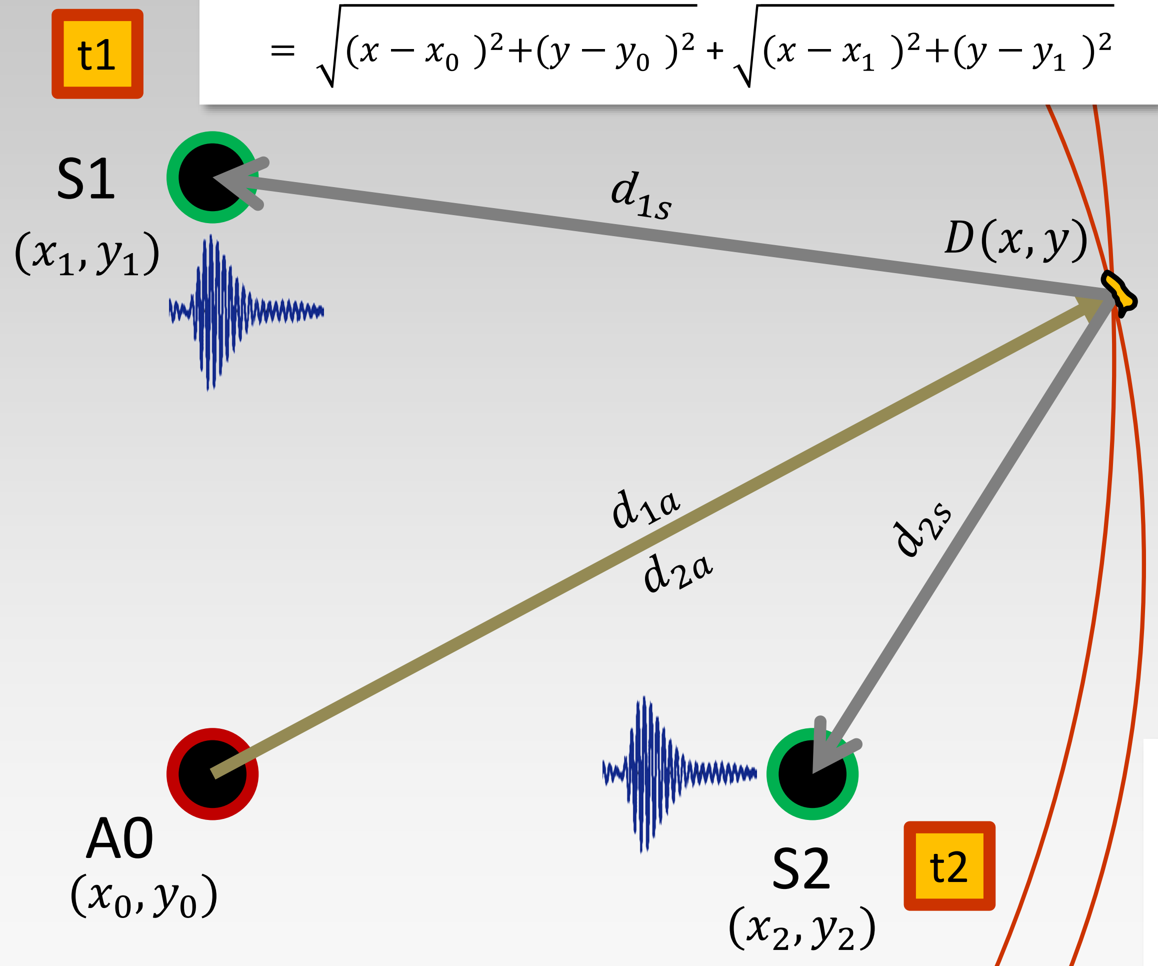
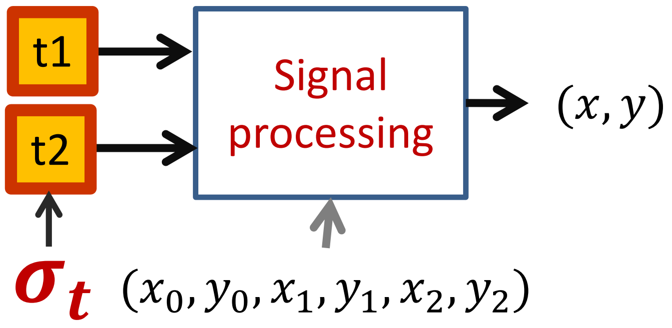


Objective of signal processing

Extract the damage coordinates *(x,y)* from t1 & t2



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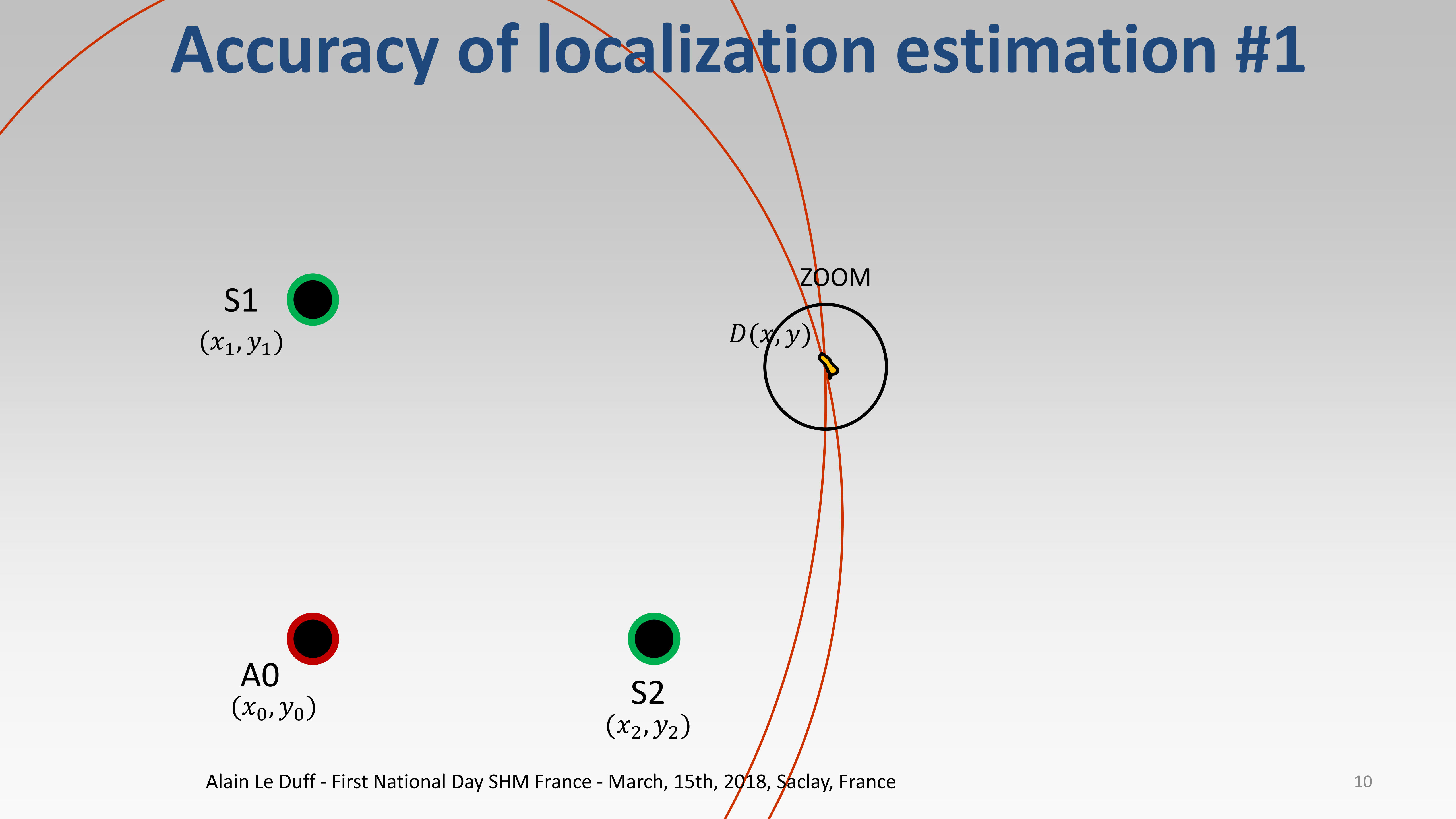
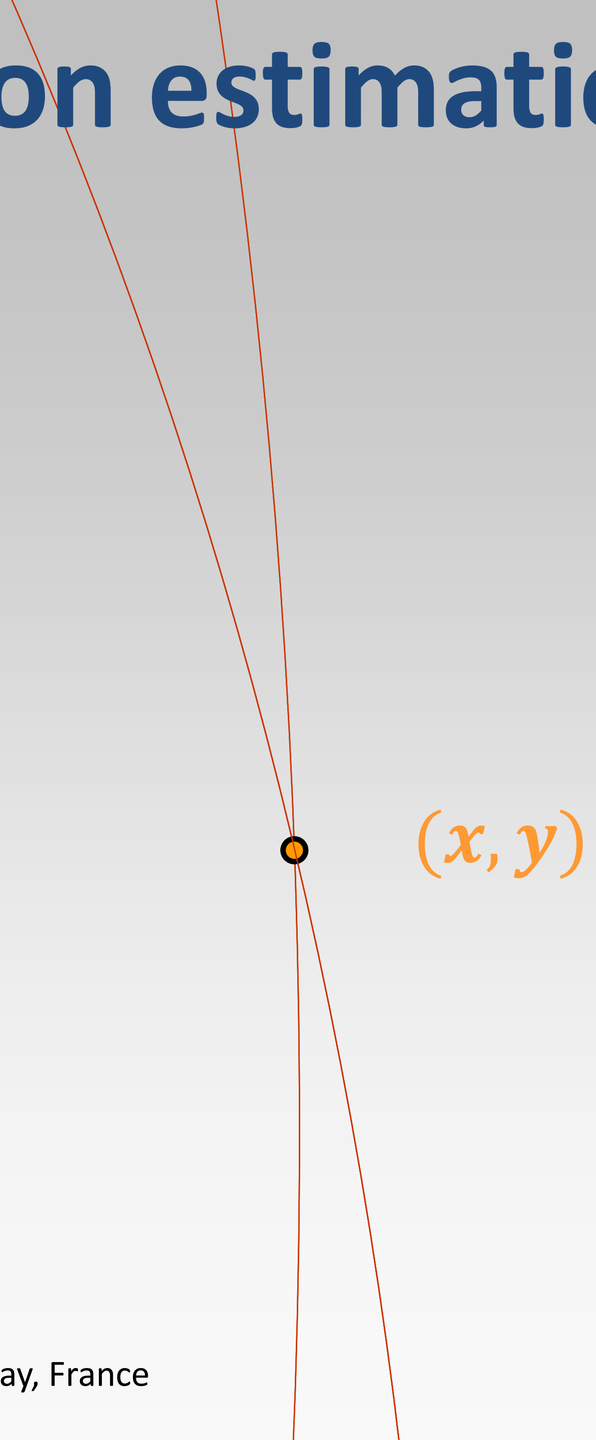


Accuracy of localization estimation #2

Errors in time delay estimation

↓

error on damage position estimation



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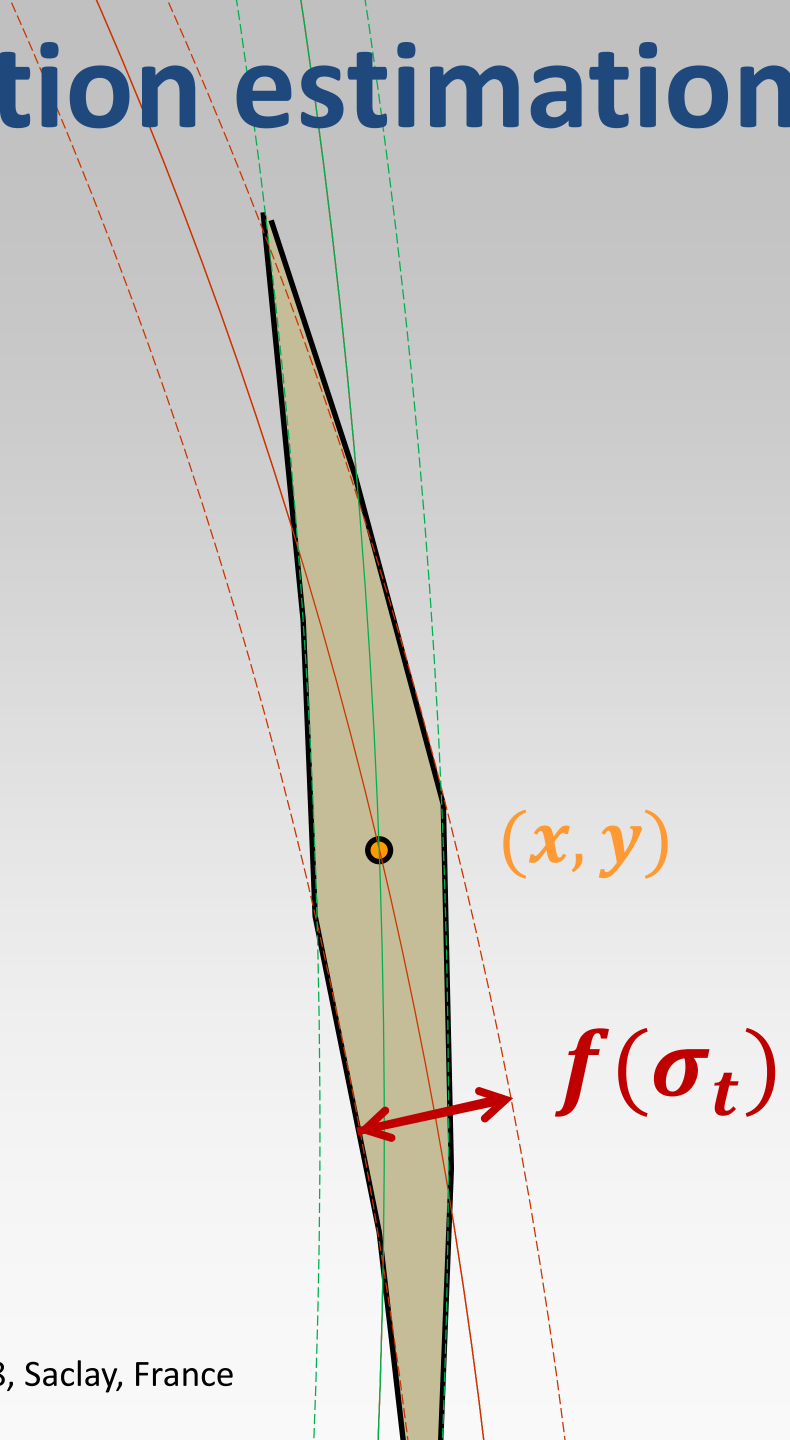

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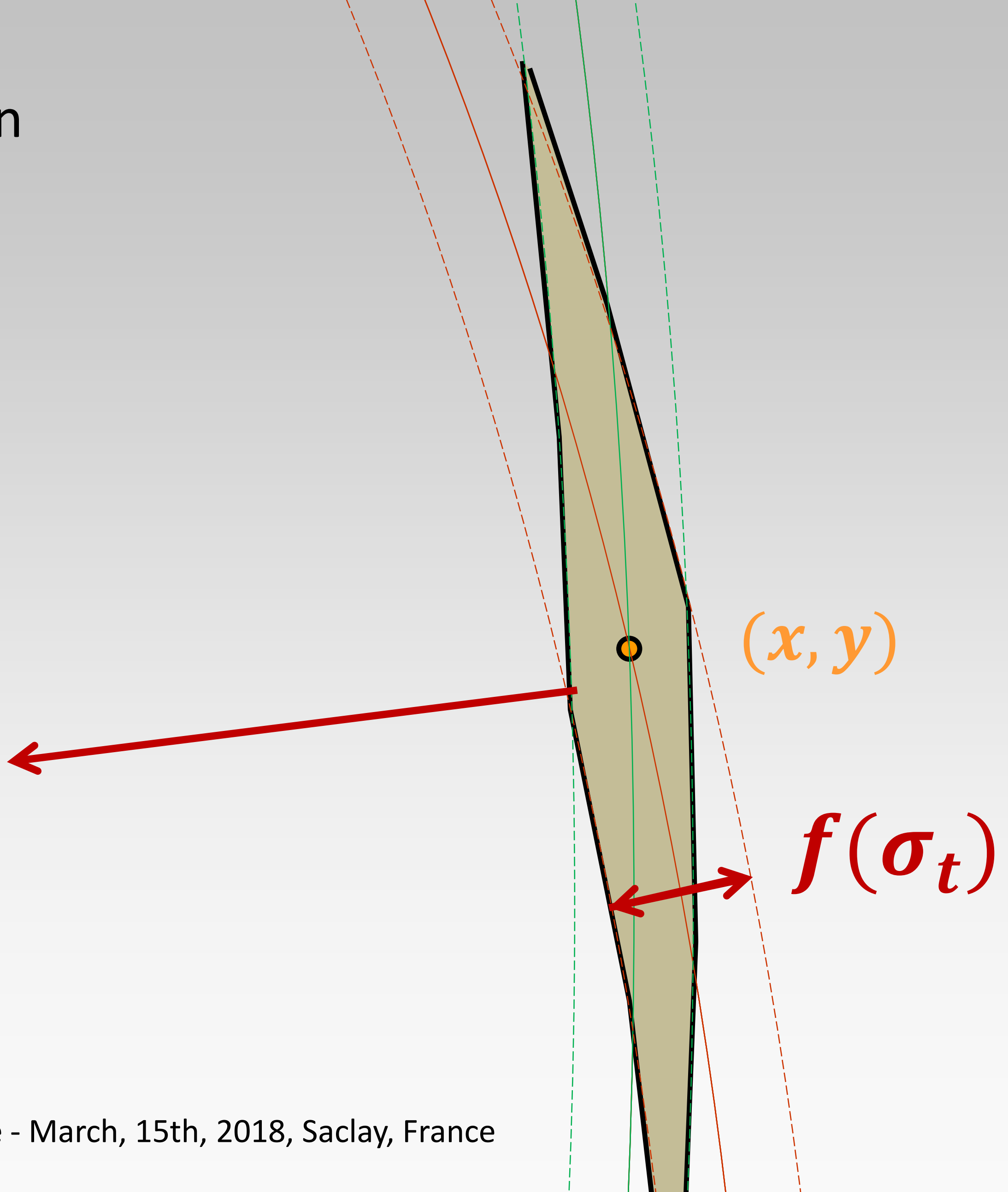
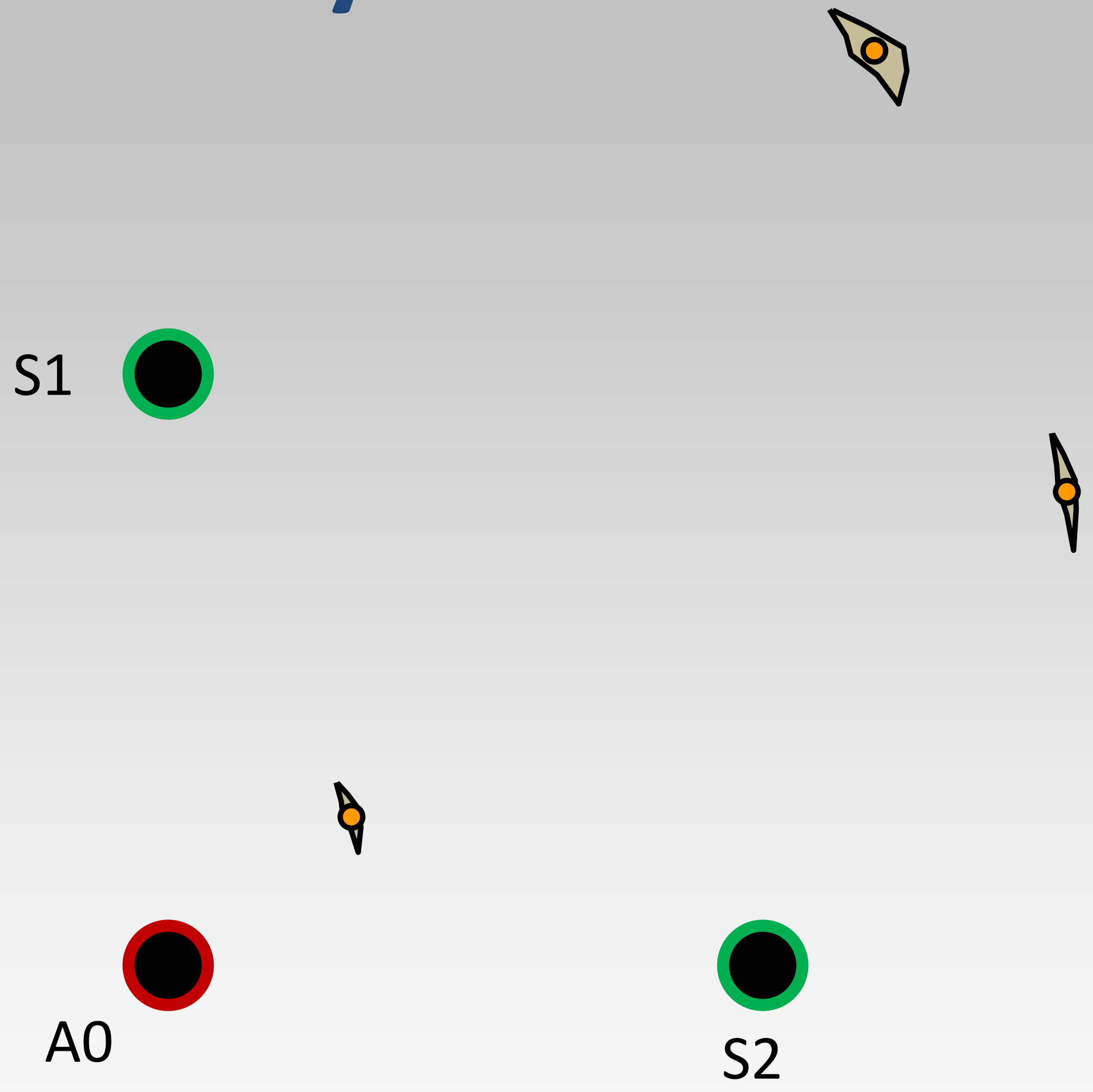
Uncertainty area (extend of error)

↓

Accuracy of the time delay measurement $\sigma_t{}^2$ (estimation variance)



Accuracy of localization estimation #3



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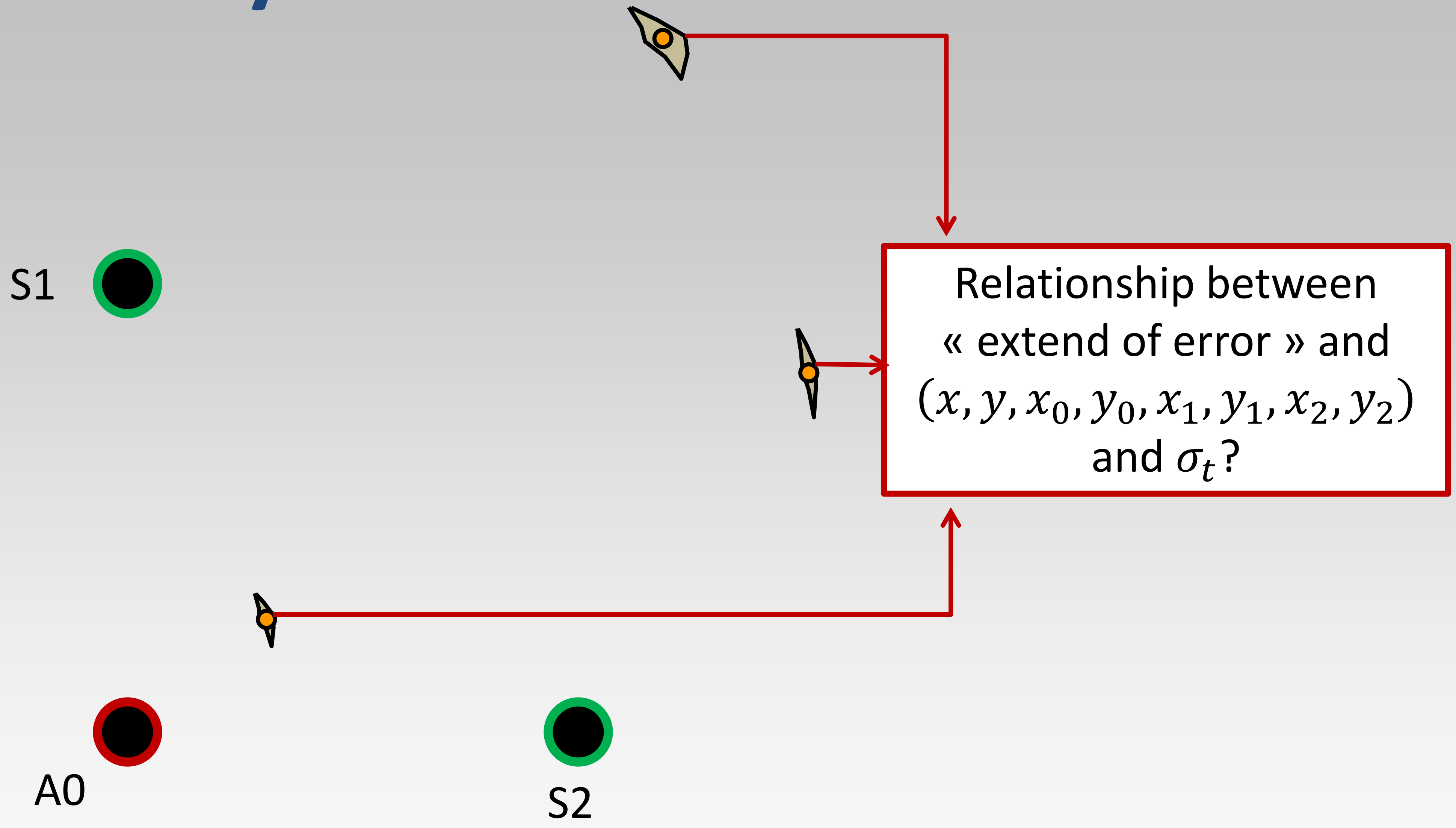
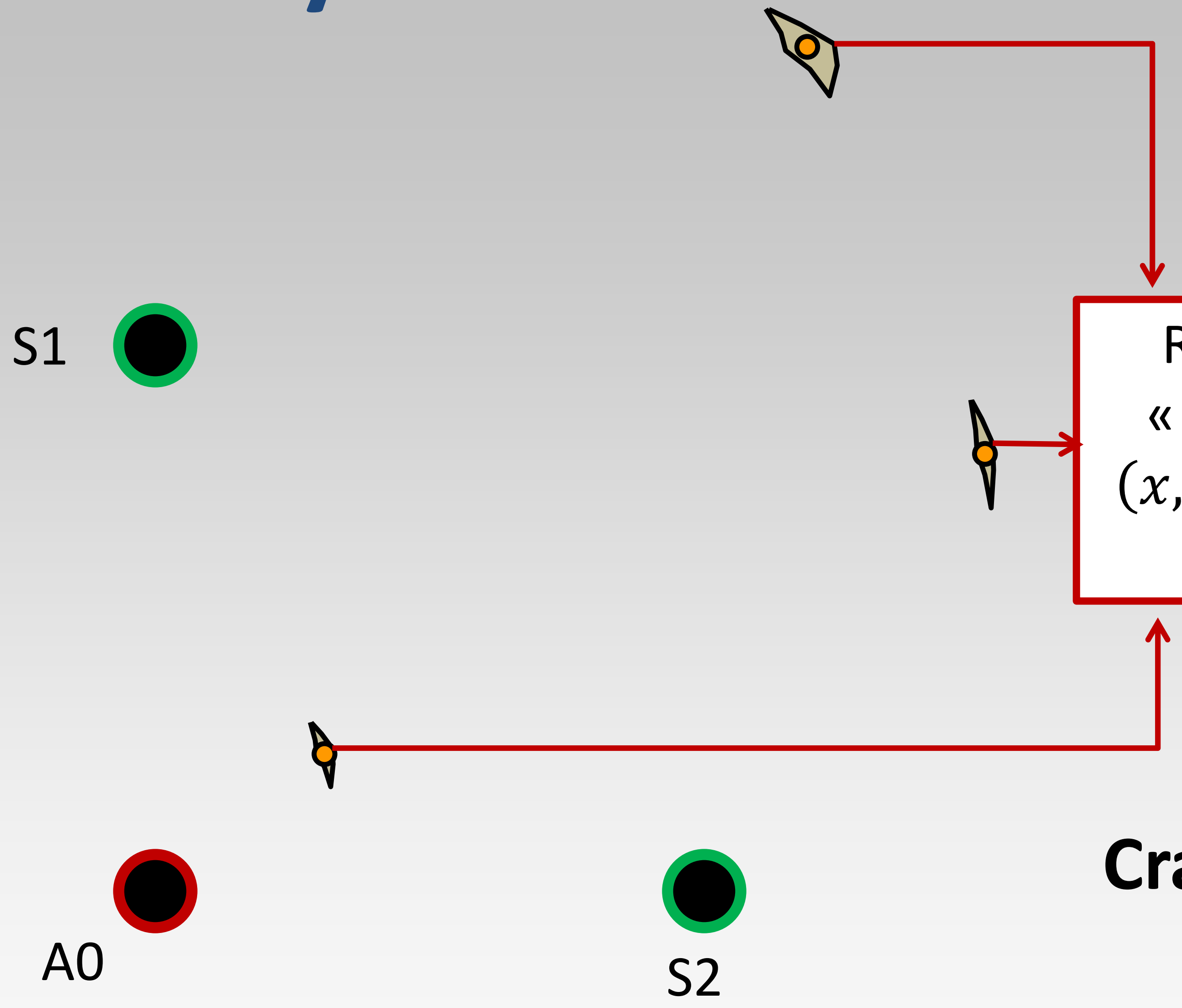


Accuracy of localization estimation #3



Relationship between « extend of error » and $(x, y, x_0, y_0, x_1, y_1, x_2, y_2)$ and σ_t?

Cramer-Rao Bounds (CRB)

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- Give a lower bound for the covariance matrix of any unbiaised estimator

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Cramer-Rao Bounds (CRB)

- Give a lower bound for the covariance matrix of any unbiaised estimator
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- Provide a benchmark against which the performance of estimators can be compared
- In this work: the CRBs give an indication on the *a priori* accuracy of the localization procedure

Exact analytical CRB #1

Observed data: $t_1 = \tau_1 + n_1$ $\quad t_2 = \tau_2 + n_2$

Hypothesis : n_1 and n_2 are independant White Additive Gaussian Noises (WAGN)

Time delay estimations variances

$$\sigma_{t_1}^2 \propto G_1^2(x,y) \cdot \sigma_v^2$$

$$\sigma_{t_2}^2 \propto G_2^2(x,y) \cdot \sigma_v^2$$

Additive noise variance

$$G_1(x,y) = \sqrt{d_{1a} d_{1s}} \qquad G_2(x,y) = \sqrt{d_{2a} d_{2s}}$$

Cylindrical attenuation

$$\boxed{\mathrm{CRB}(x) = c^2 \sigma_{t_1} \sigma_{t_2} \cdot \frac{K_{21}^2 + K_{22}^2}{(K_{11} \cdot K_{22} + K_{12} \cdot K_{21})^2}}$$

$$\boxed{\mathrm{CRB}(y) = c^2 \sigma_{t_1} \sigma_{t_2} \cdot \frac{K_{11}^2 + K_{12}^2}{(K_{11} \cdot K_{22} + K_{12} \cdot K_{21})^2}}$$

with

$$K_{21} = \frac{y - y_0}{d_{1a}} + \frac{y - y_1}{d_{1s}} \qquad K_{11} = \frac{x - x_0}{d_{1a}} + \frac{x - x_1}{d_{1s}}$$

$$K_{22} = \frac{y - y_0}{d_{2a}} + \frac{y - y_2}{d_{2s}} \qquad K_{12} = \frac{x - x_0}{d_{2a}} + \frac{x - x_2}{d_{2s}}$$

Exact analytical CRB #2

In summary:

$$\mathrm{CRB}(x,y) = f\left(x, y, x_0, y_0, x_1, y_1, x_2, y_2, {\sigma_v}^2, \mathrm{c}\right)$$

Unknown damage location

Actuator and sensors location

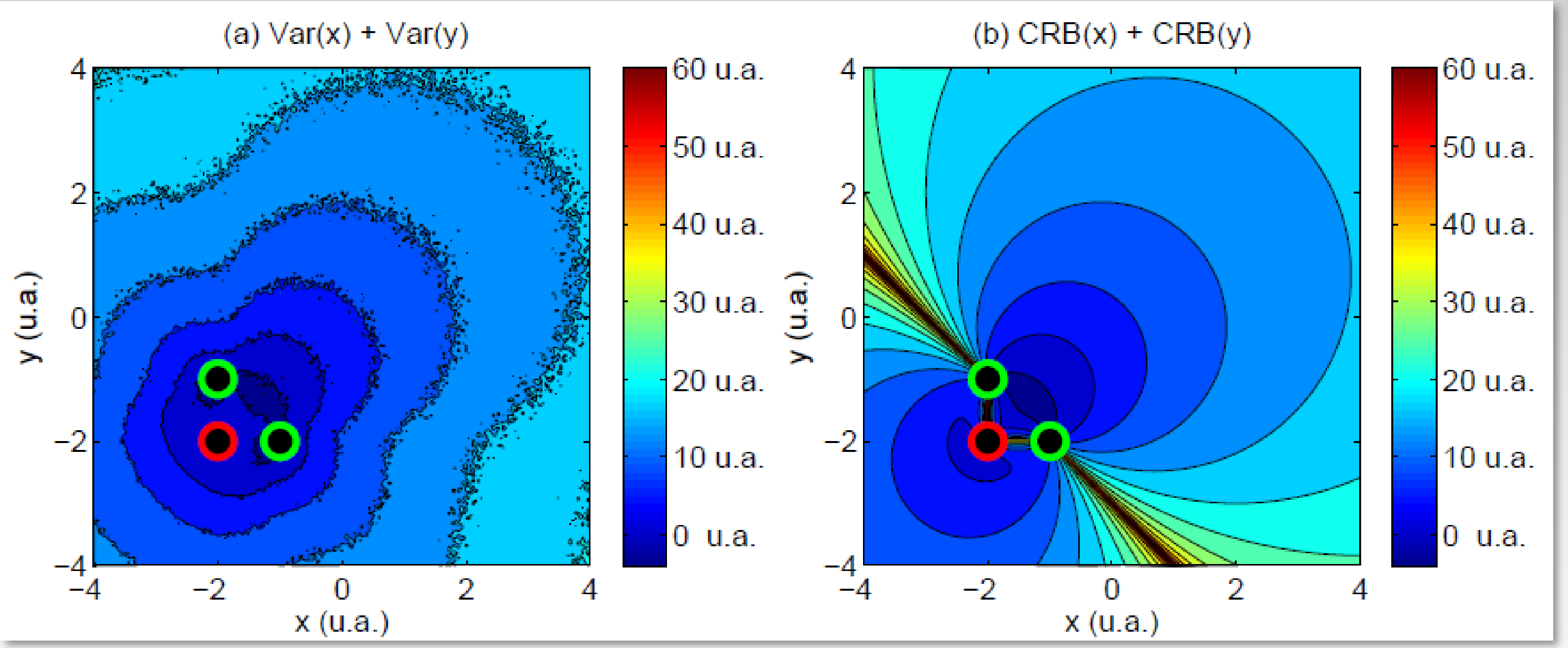
Additive noise variance

Sound velocity

Exact analytical CRB #3

Numerical validation

- Statistical performance of the procedure → illustrated by MC simulations
- Several locations of the 3 sensors



Monte-Carlo simulations

Theoretical CRBs

Exact analytical CRB #3

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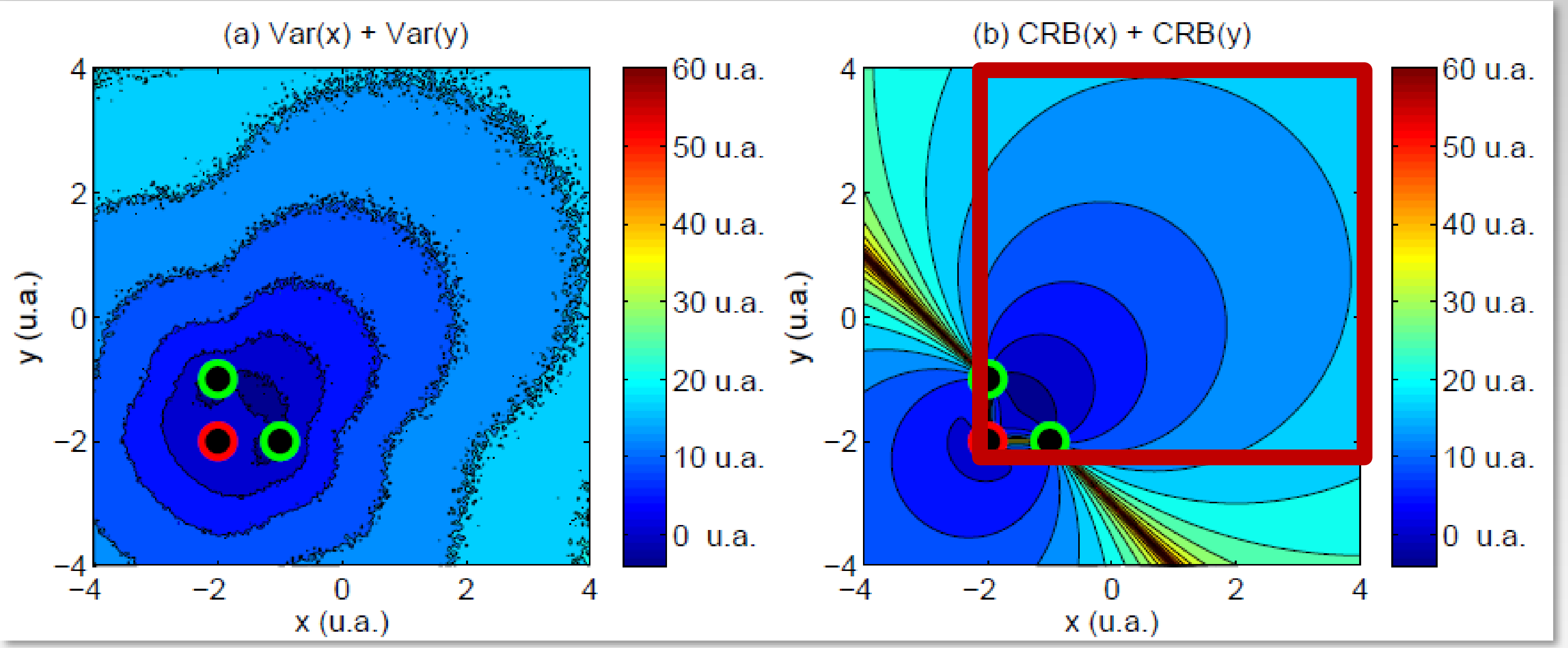

Monte-Carlo simulations

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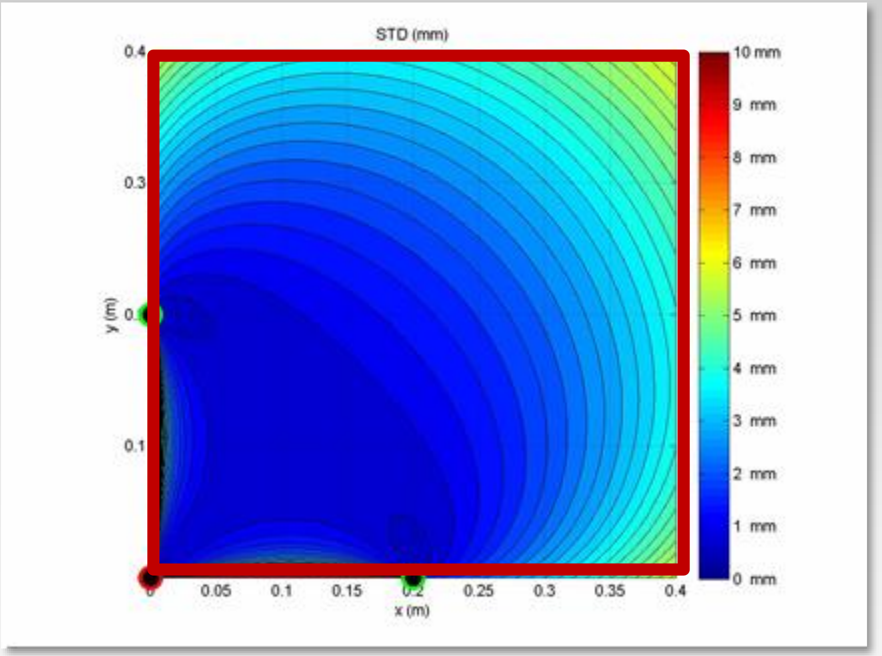
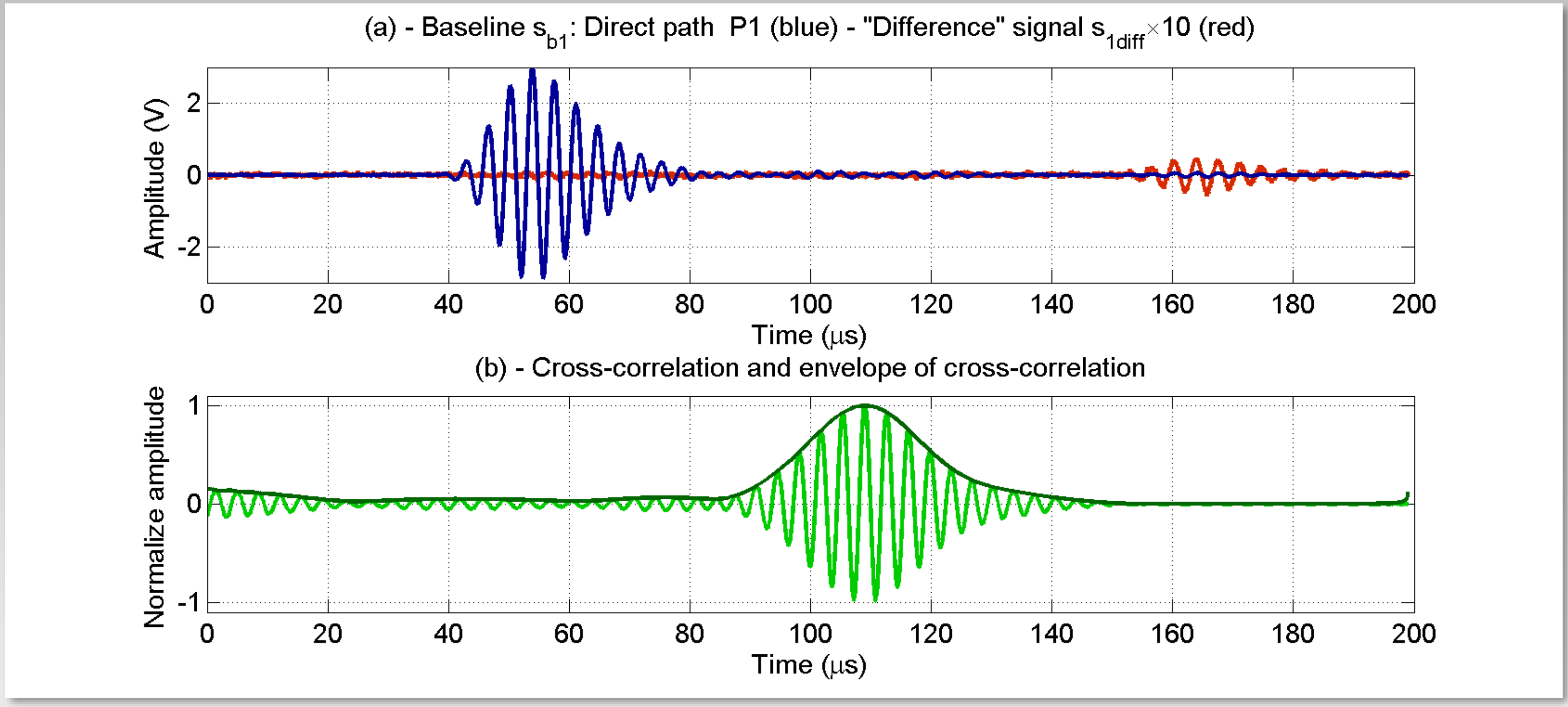
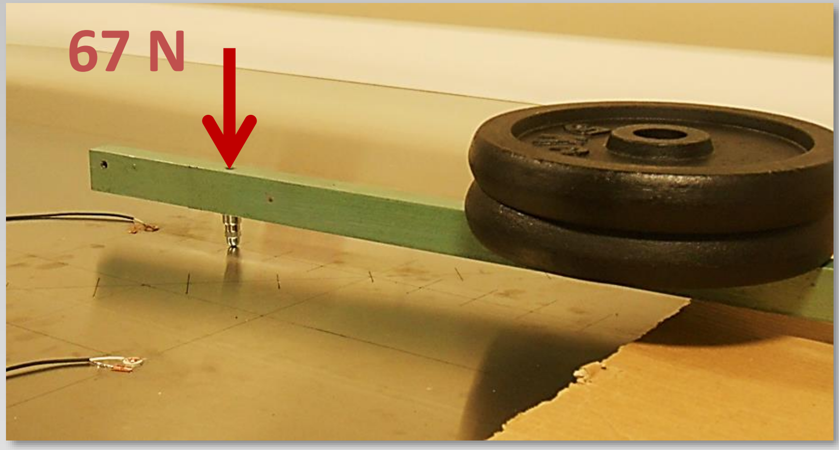
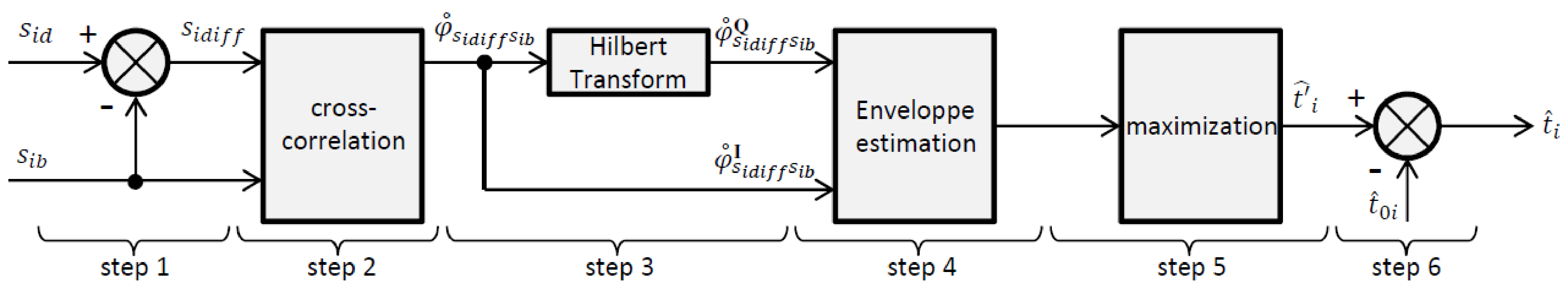


Experimental results #1





Principle of TOF measurement



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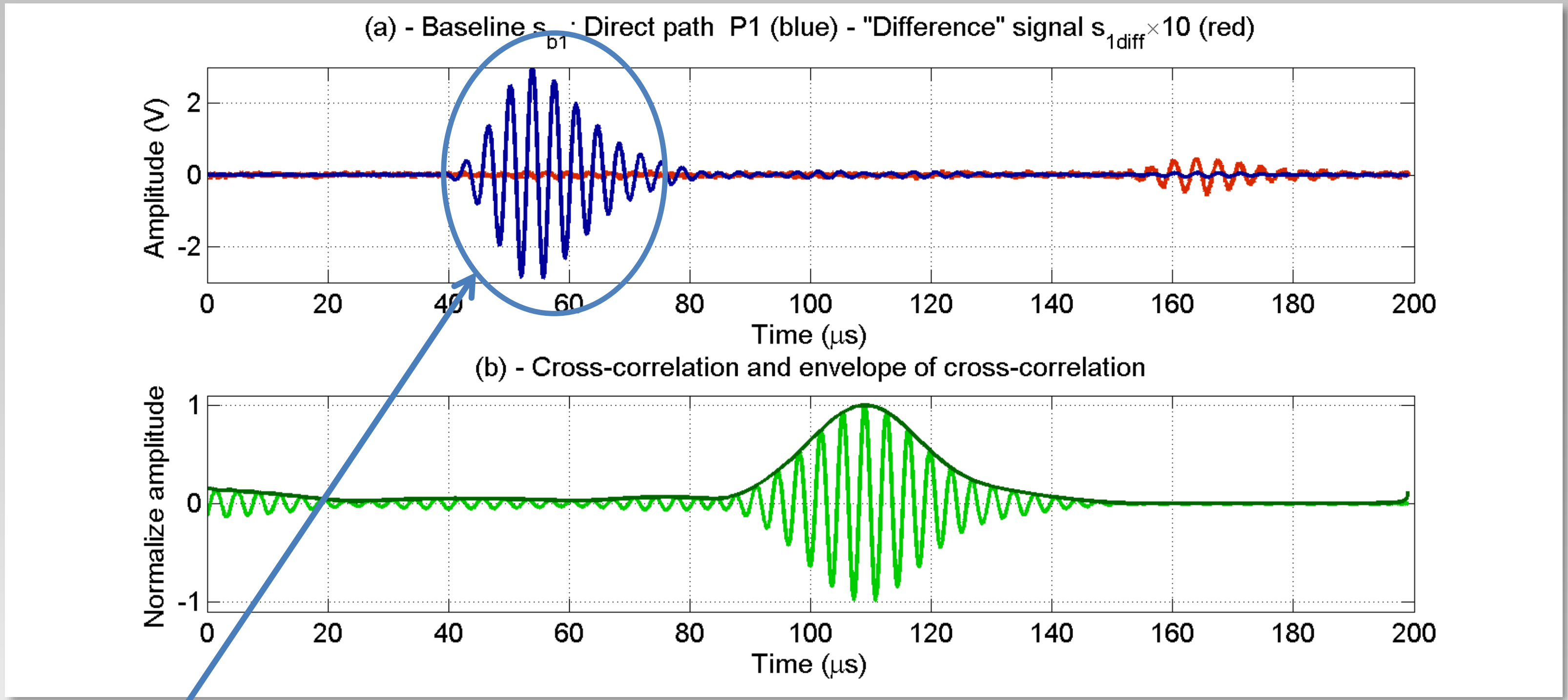

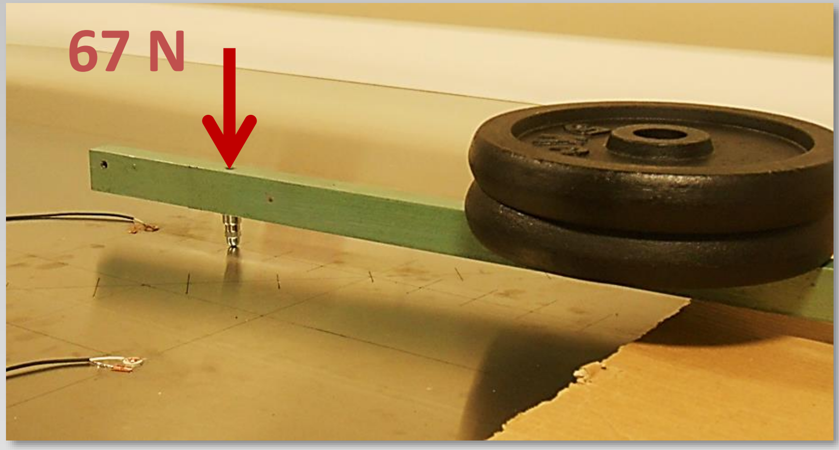

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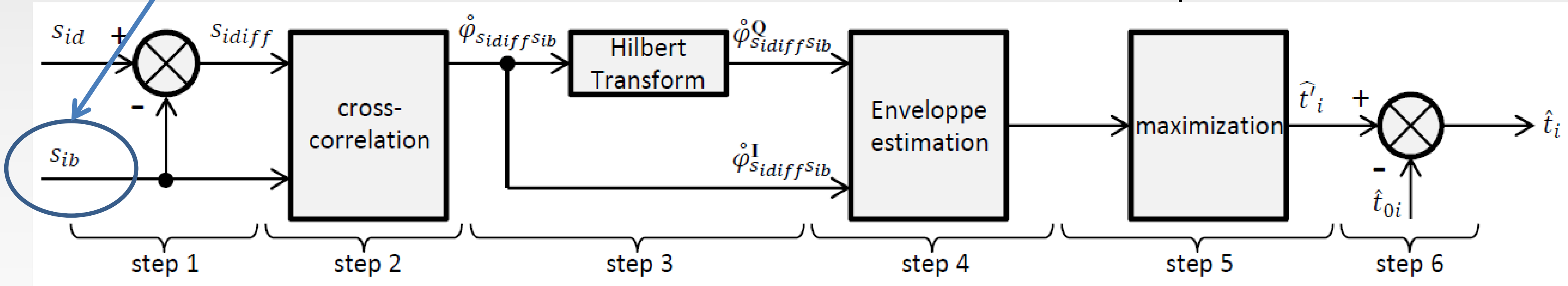

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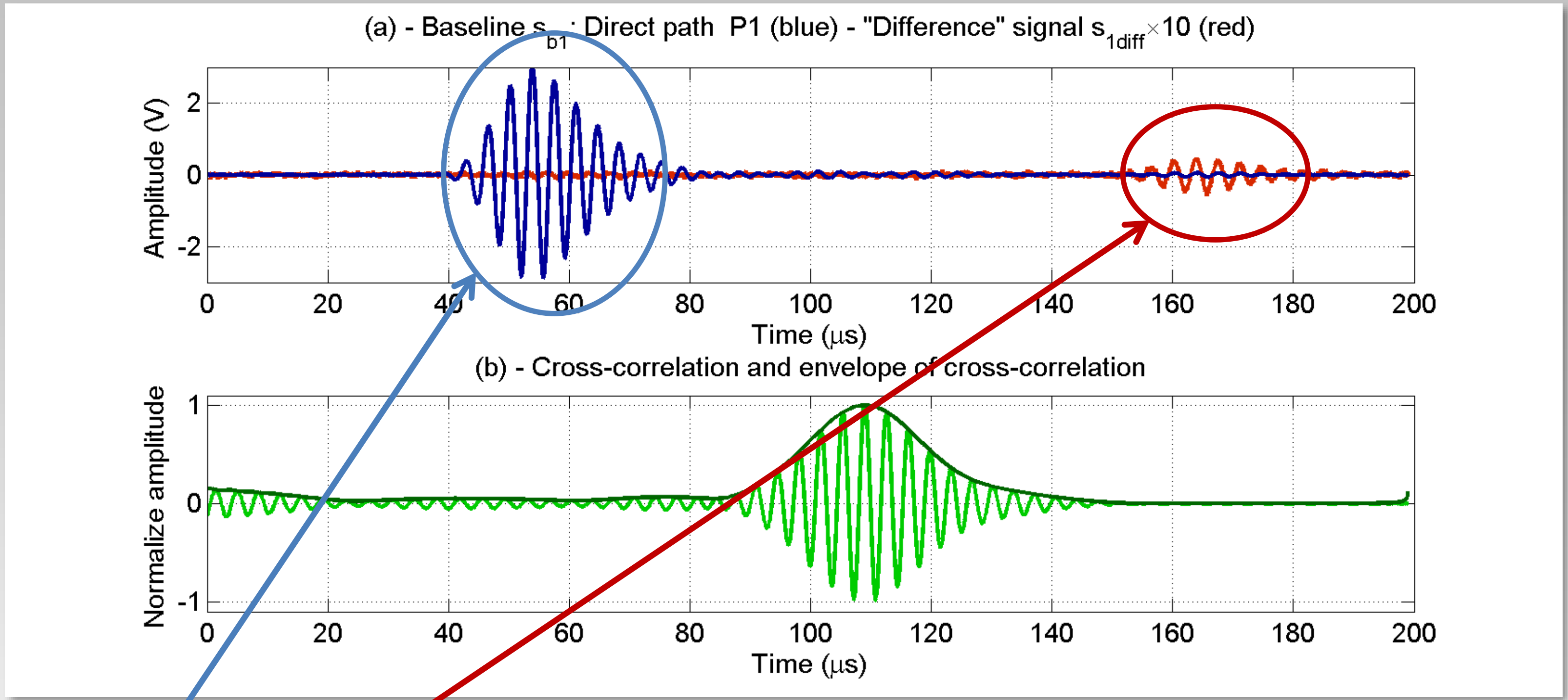

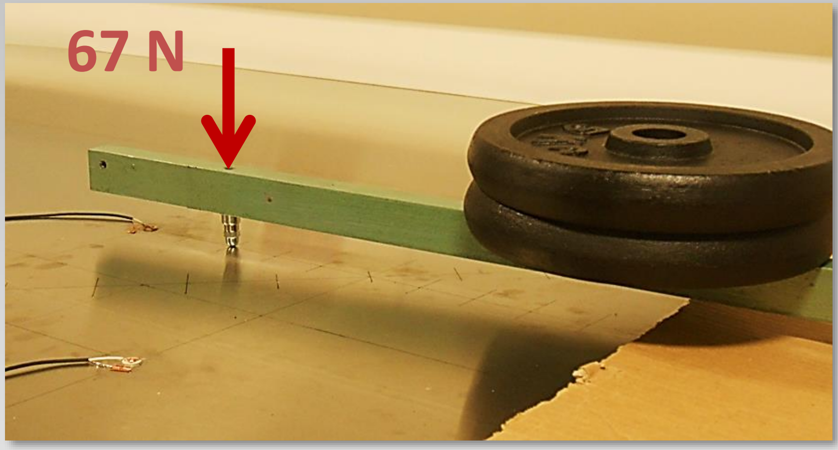

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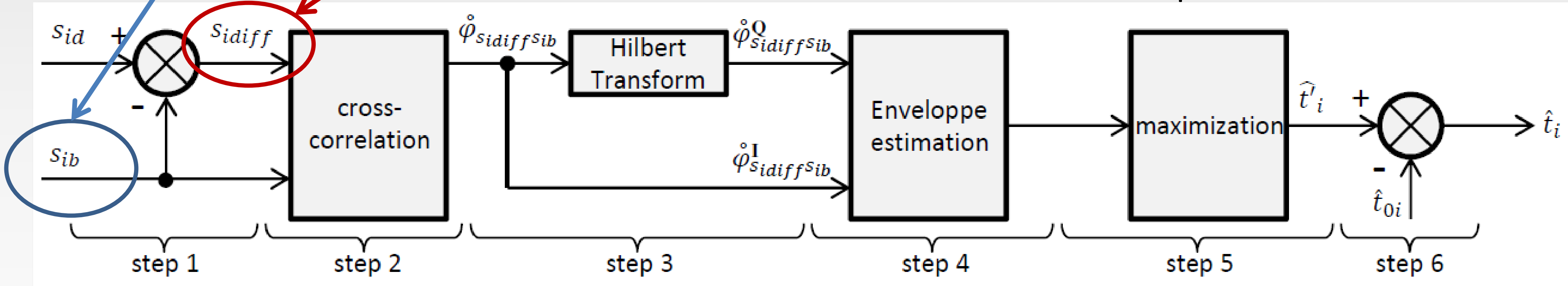

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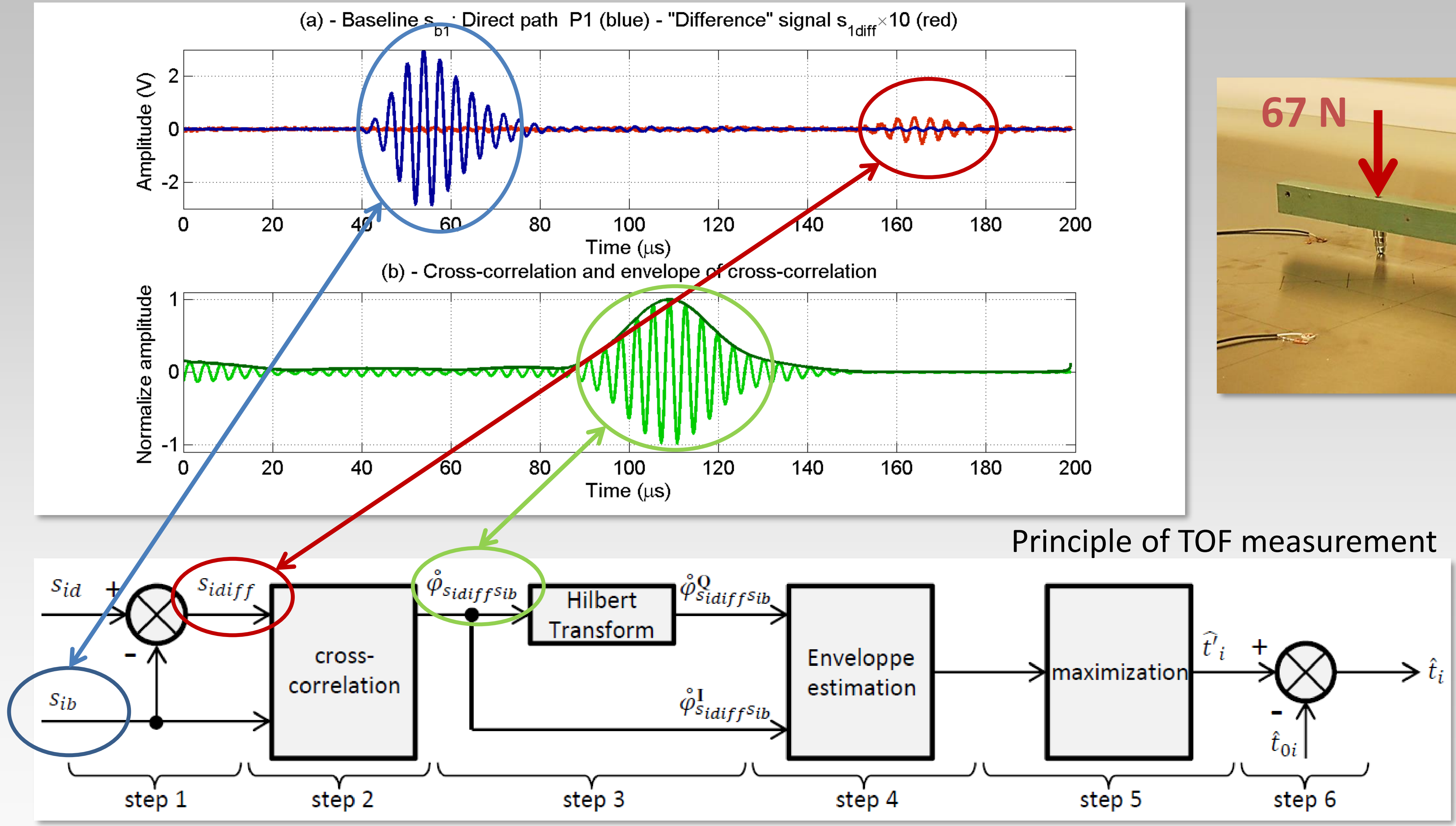
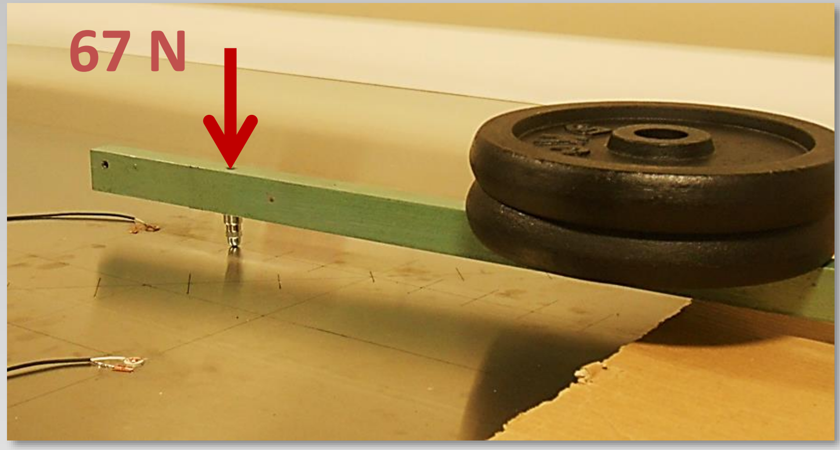
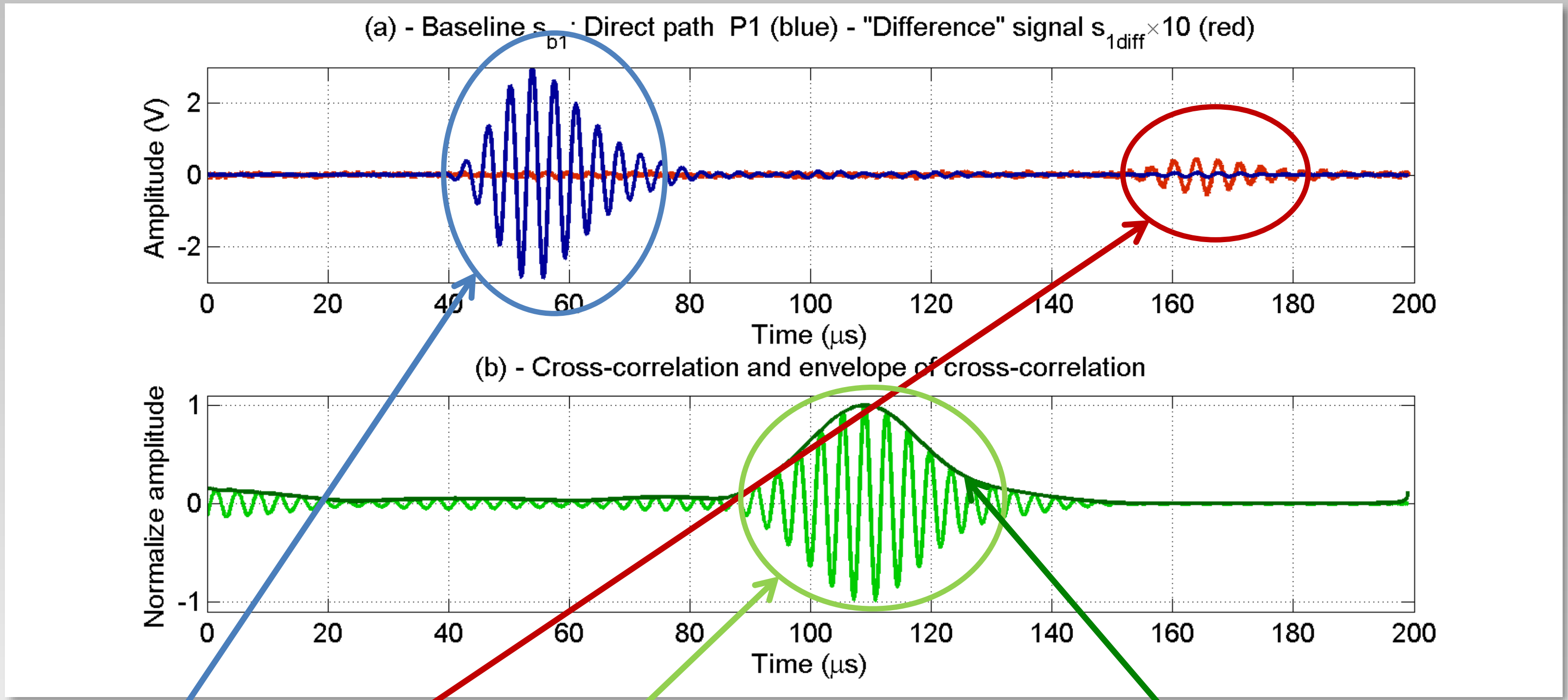
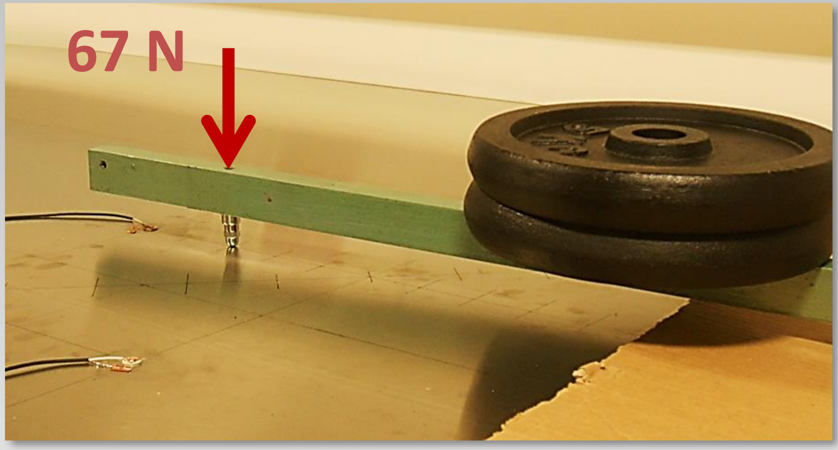


Principle of TOF measurement

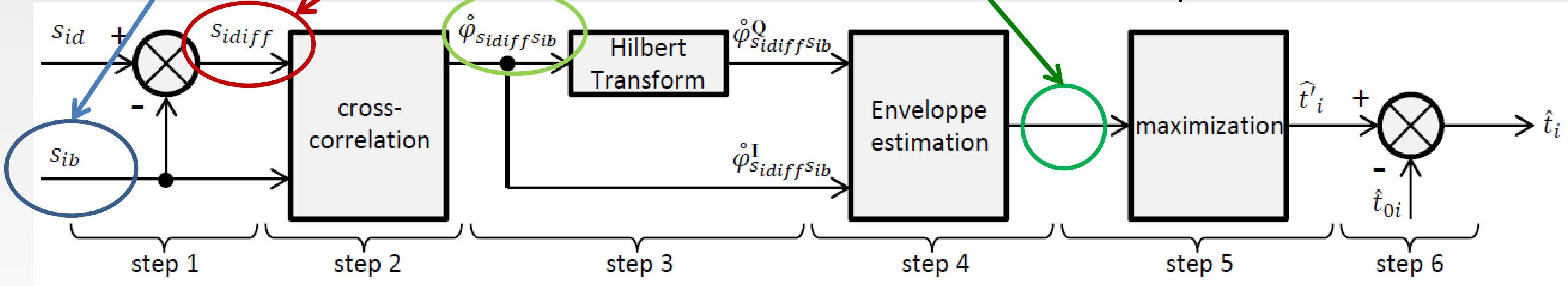


Experimental results #1

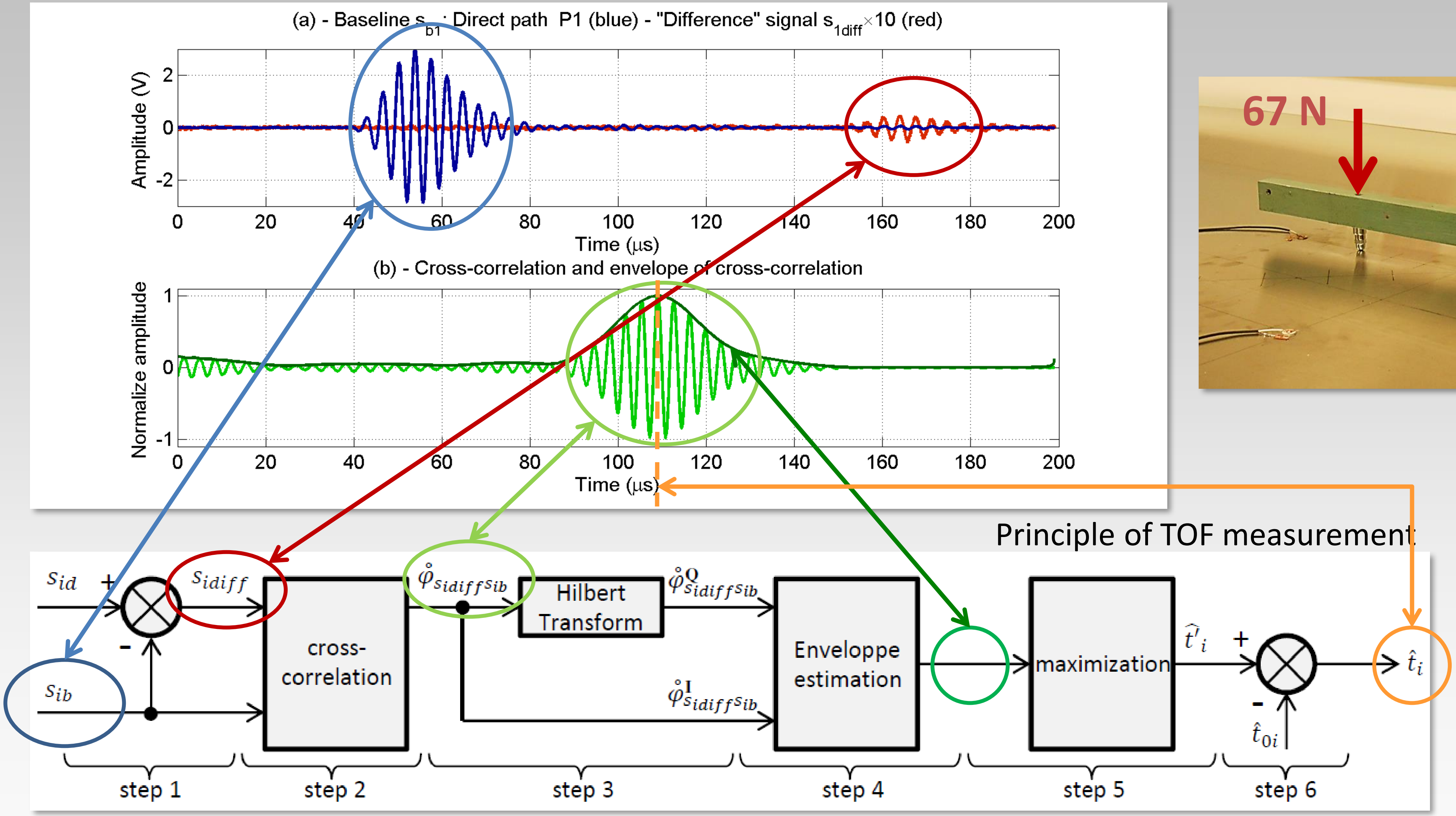


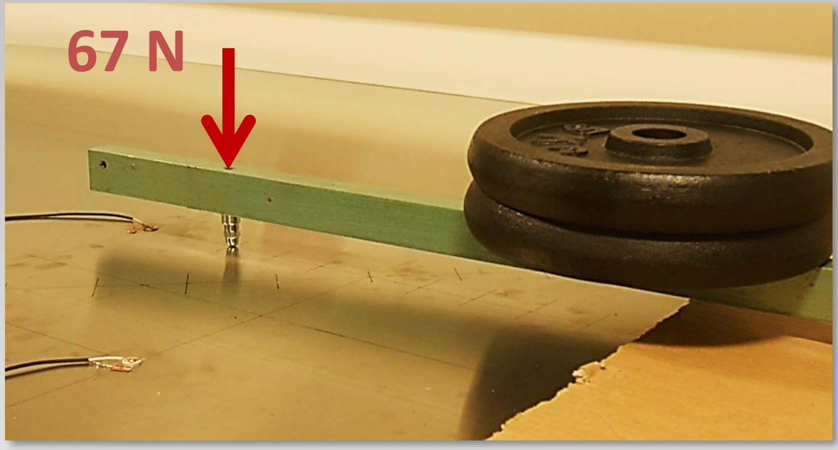


Principle of TOF measurement

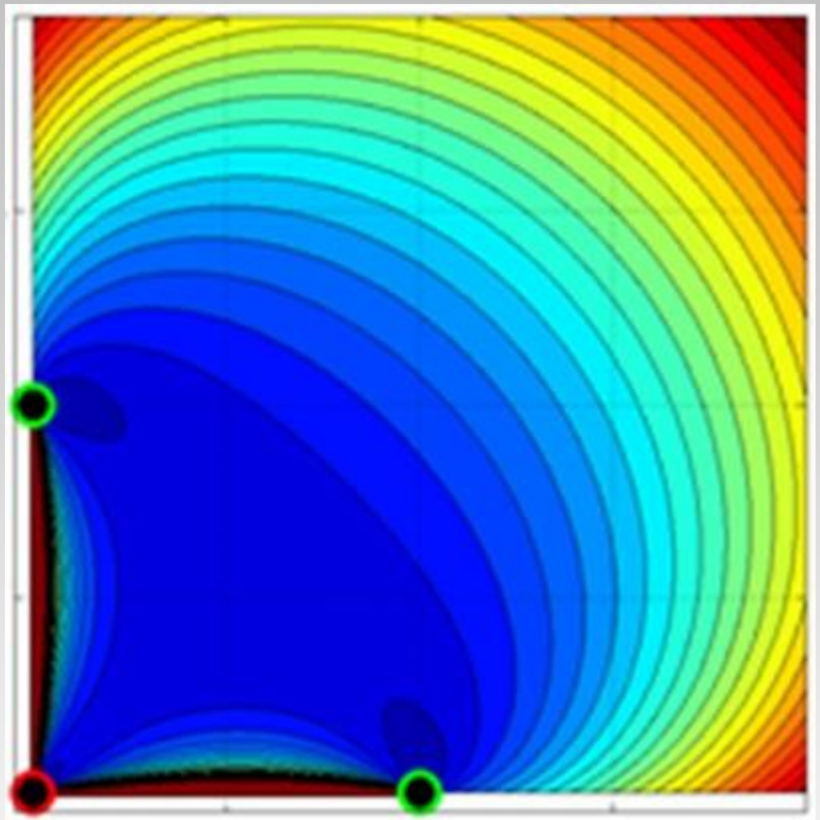


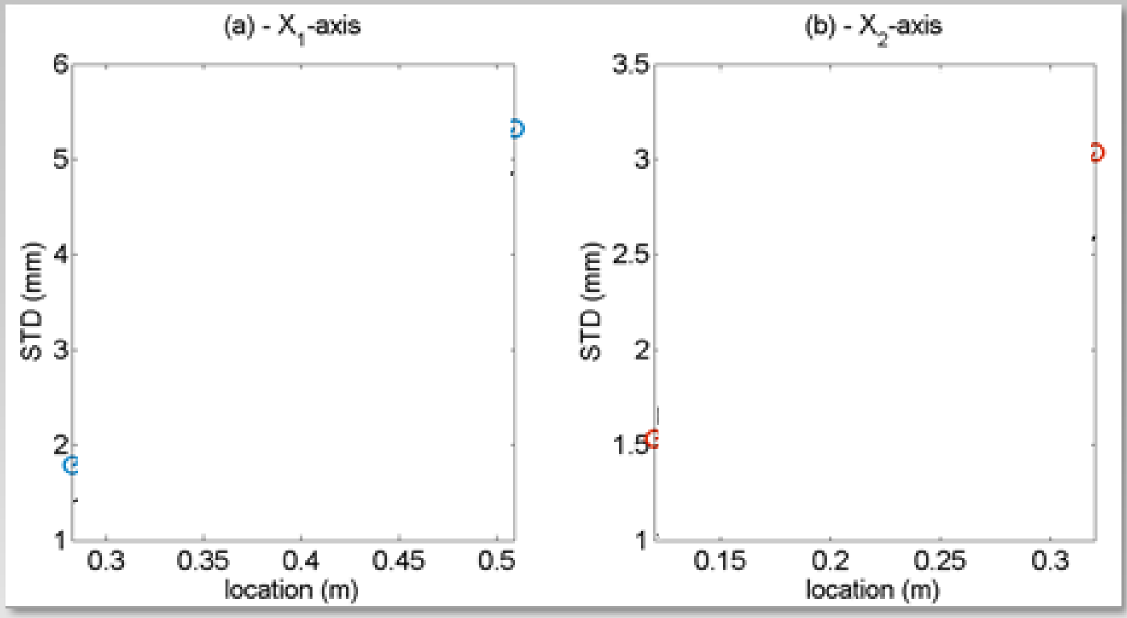
Experimental results #1





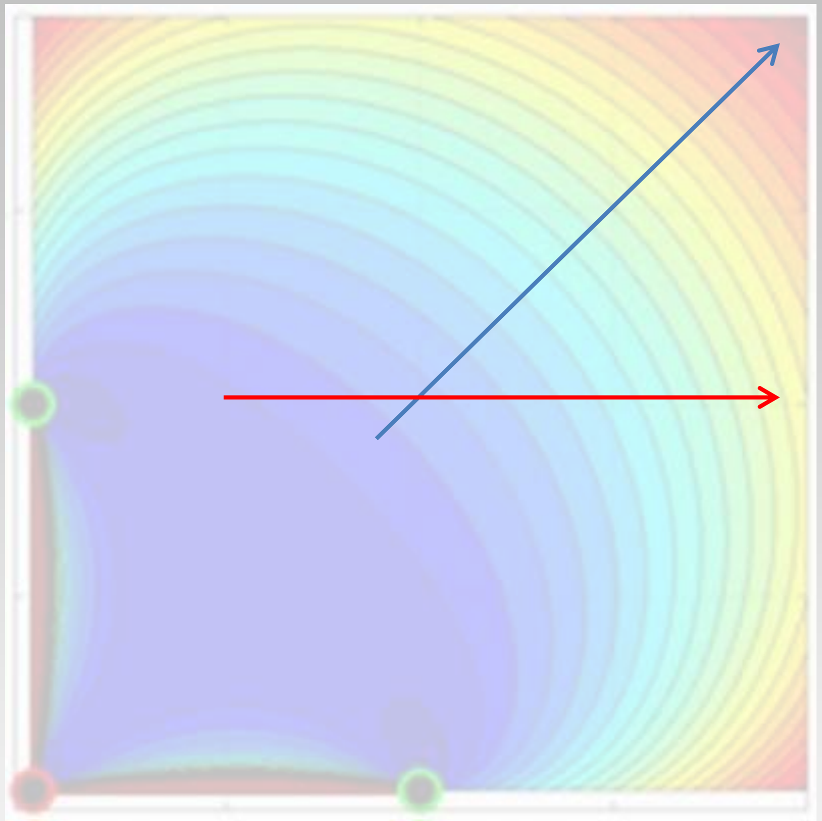
Experimental results #2

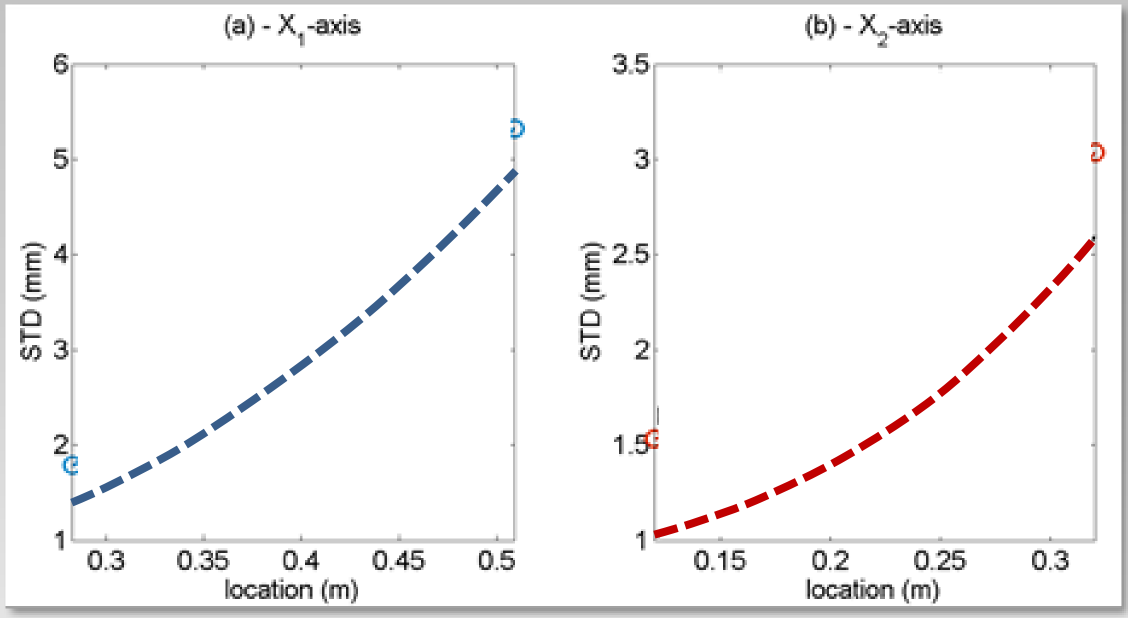




- Theoritical STD
- Experimental STD

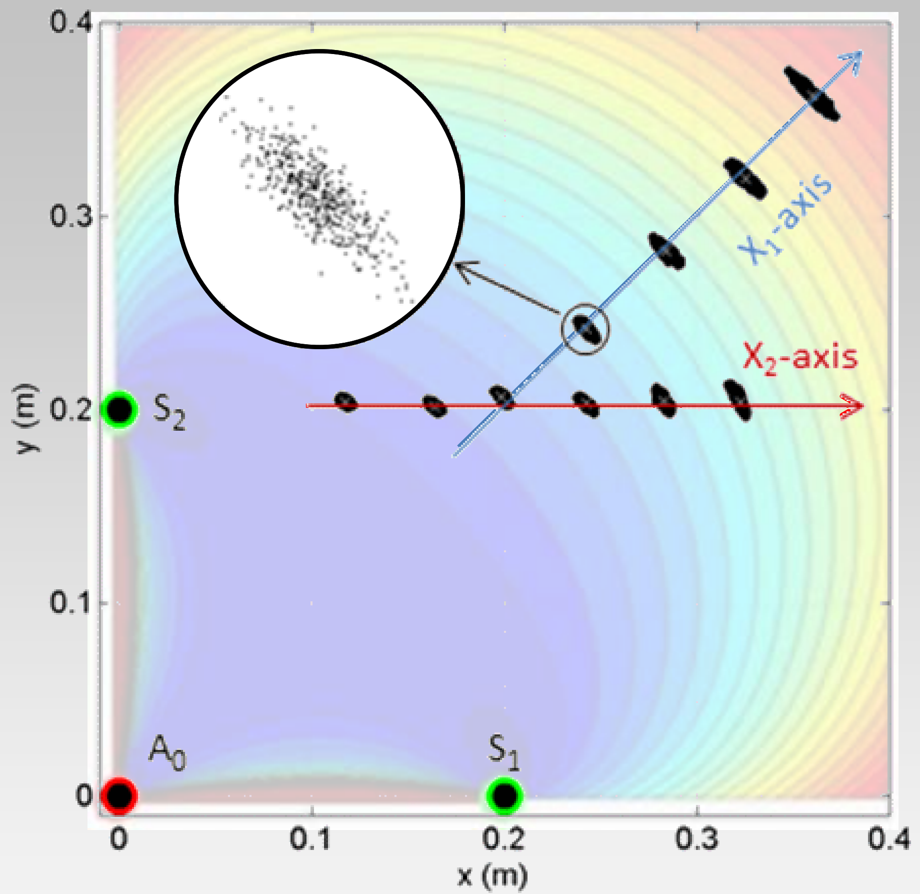
Experimental results #2

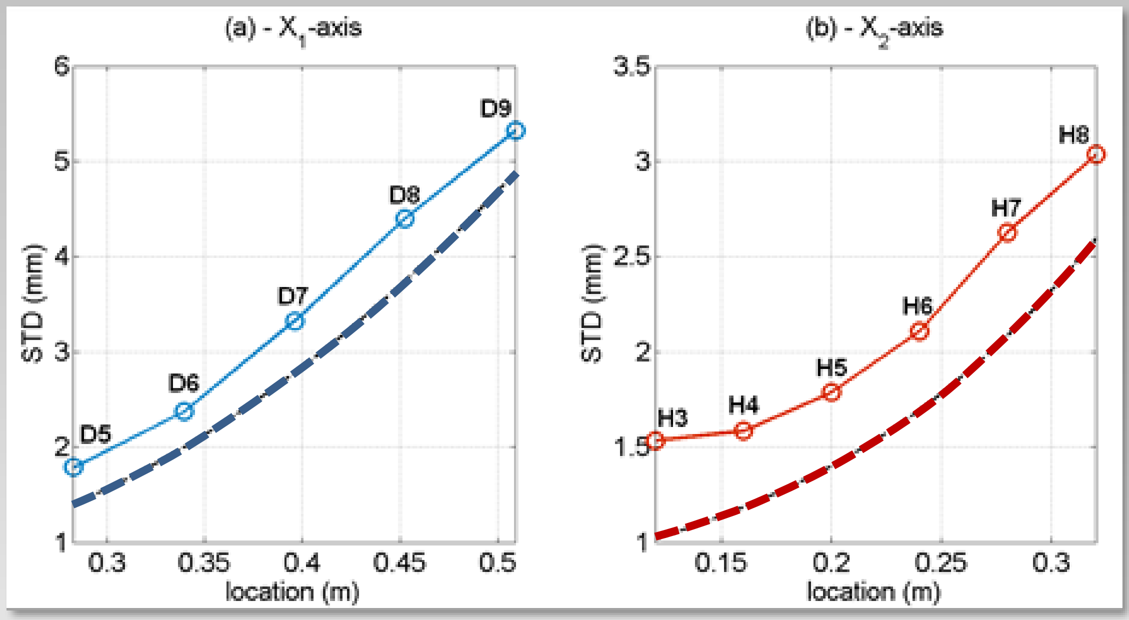




- Theoritical STD
- Experimental STD

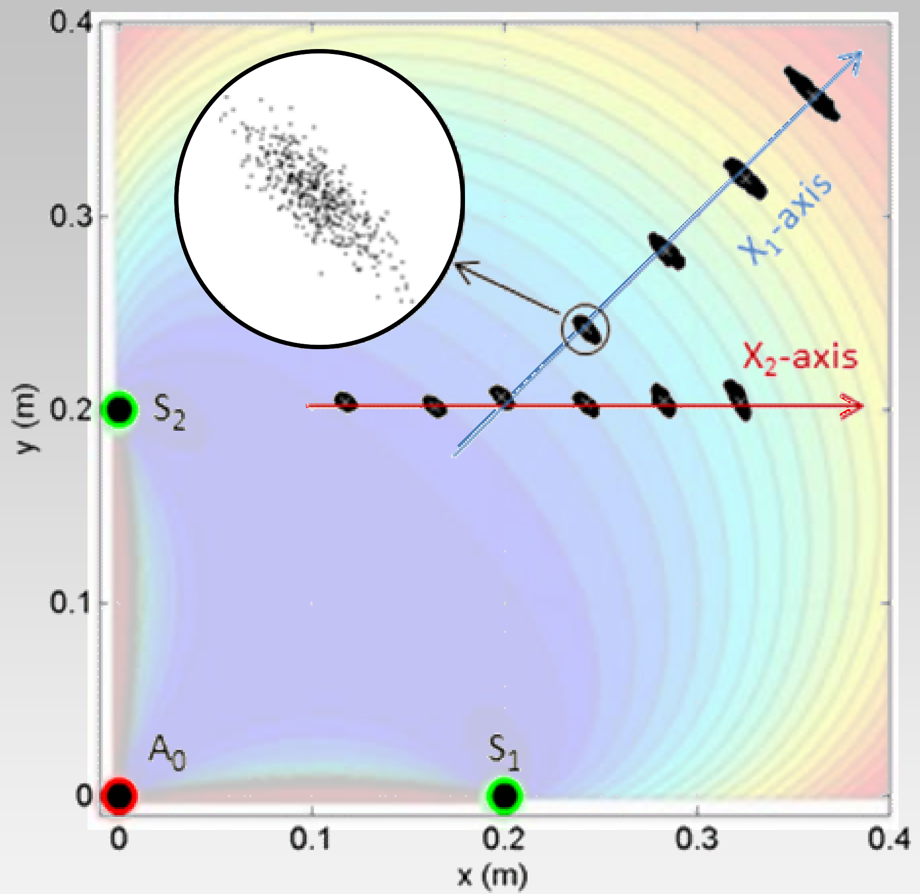
Experimental results #2

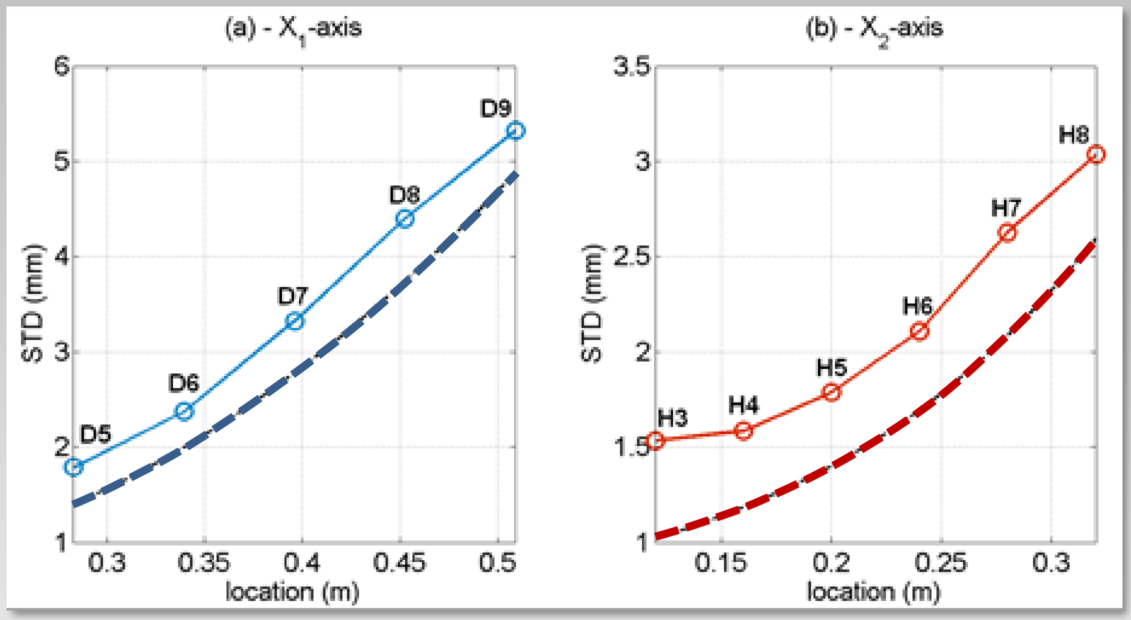




- Theoritical STD
- Experimental STD

Experimental results #2



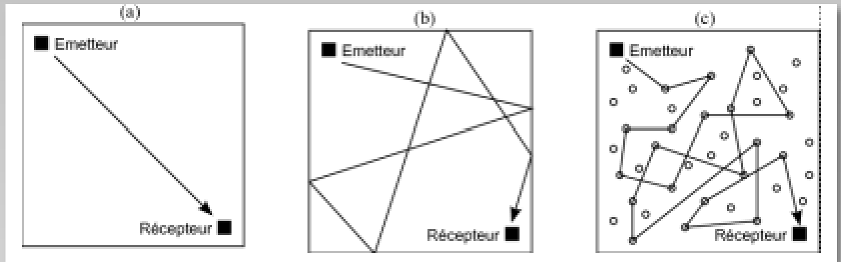


- Theoritical STD
- Experimental STD

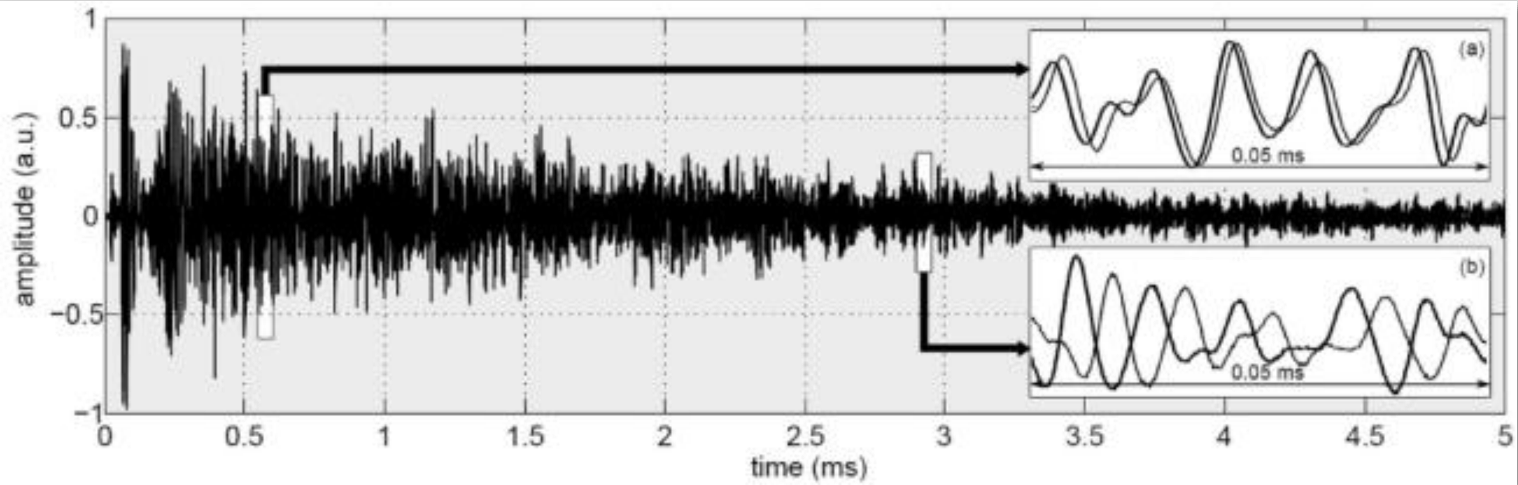
- Performance of the estimations follows the theoretical STD
- CRBs → good indication of the expected accuracy of a particular configuration of a SHM system
  - as a function of the geometry of a transducers array;
  - and SNR of the data acquisition.

Example #2: temperature estimation

- **Context**
  - Temperature estimation
  - Scale factor estimation → CWI (Coda Wave Interferometry)



- **Proposition**:
  - Study of temperature change → CRB
  - Experimental assessement of CRB using GW



Example : θ change $x_0(t) \rightarrow x_1(t) \approx x_0(\alpha t)$ → **θ estimation**

T0 *T1*

« Historical » estimators: cross-correlation - Stretching

Scale transform based estimator

Work done

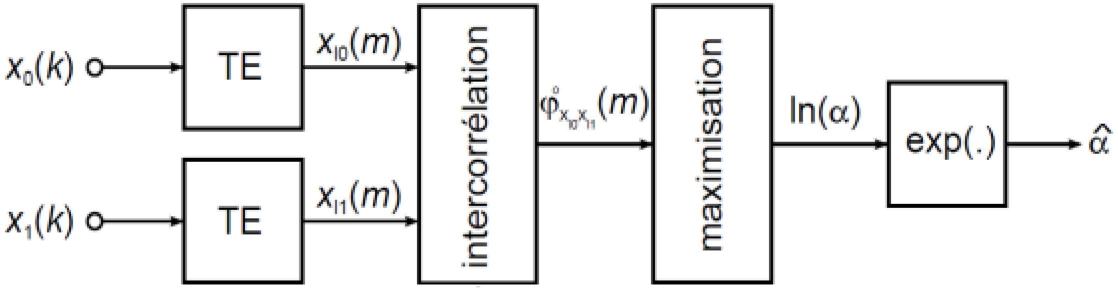
1. Study of 4 scale factor estimators (including 2 original)

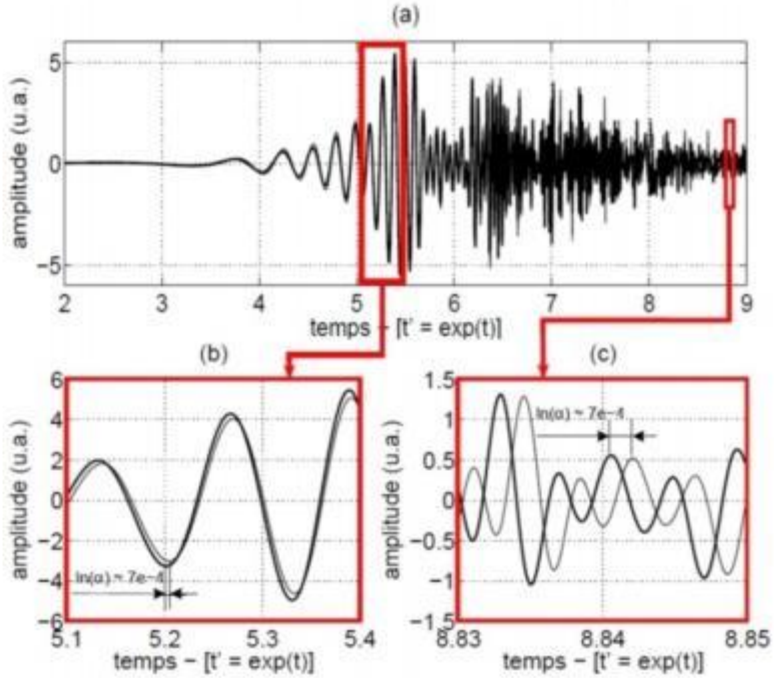
$$x_1(t) = x_0(\alpha t)$$

Exponential transform

$$x_{l_1}(t) = \frac{1}{\sqrt{\alpha}} x_{l_0}(t + \ln \alpha)$$

Time delay

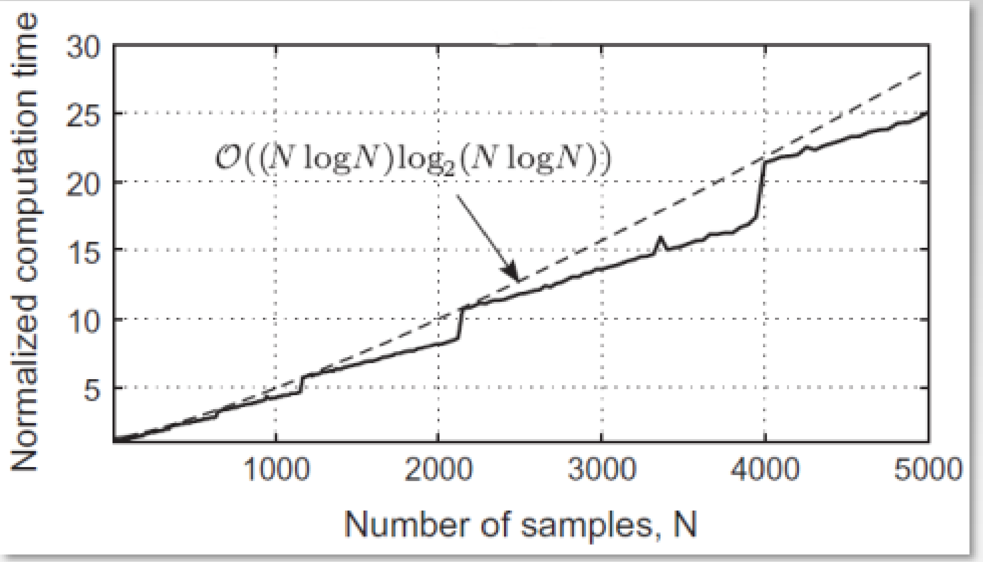


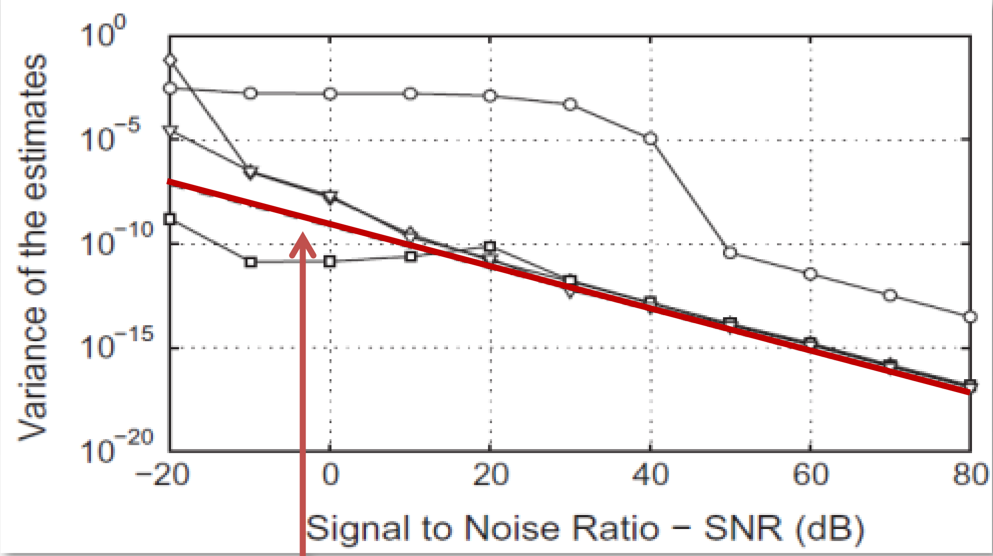


Algorithmic complexity and CRB

Work completed

2. Algorithmic complexity study
3. Analytical CRB: as a function of the testing signal parameters and SNR
4. Validation → Monte-Carlo simulation





$$CRB(\alpha) \approx \frac{1}{12\pi^2} \cdot \frac{\alpha^2}{DT_s f_0^2 \tau^2} \cdot \frac{1}{\mathrm{SNR}}$$

Experimental validation #1

Application to aluminum

Work completed (cont'd)

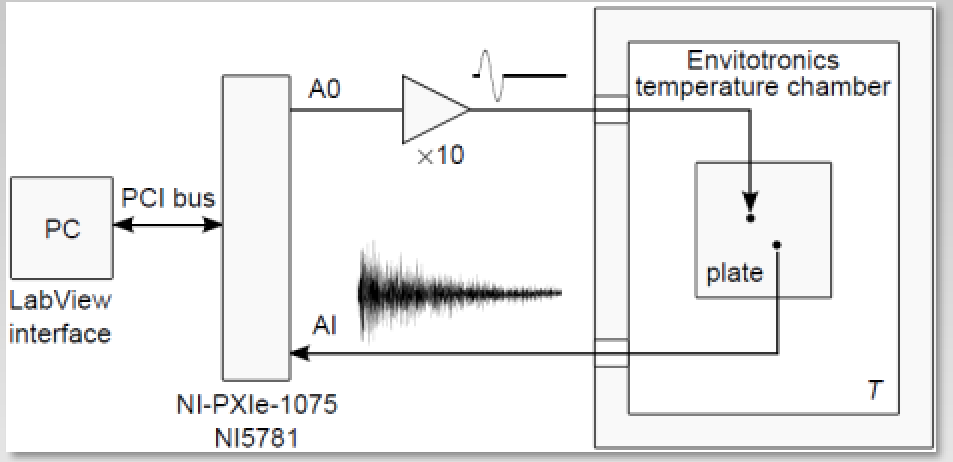
5. Experimental validation (aluminum plate)

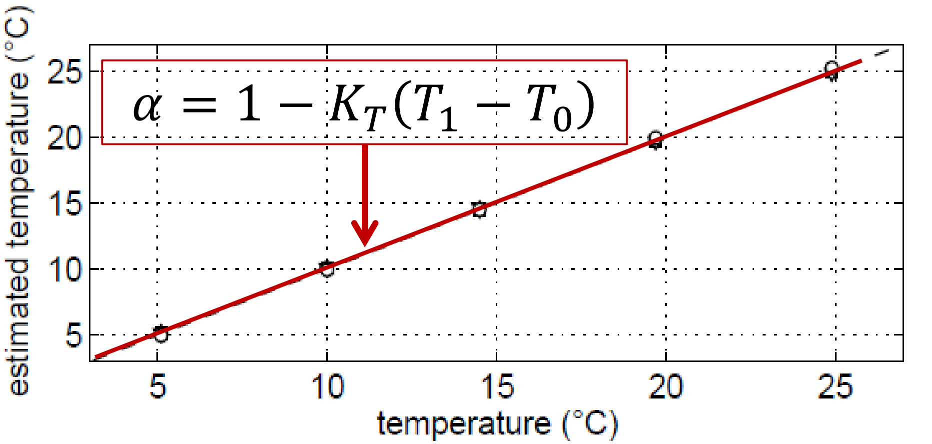
Contribution to the scientific community

- CRB → influence of parameters on accuracy
- Methods comparison
  → selection guide

Other works

- Application to composite materials
- Temperature compensation
  - Concrete (IFSTTAR)
  - Aluminum (GAUS)

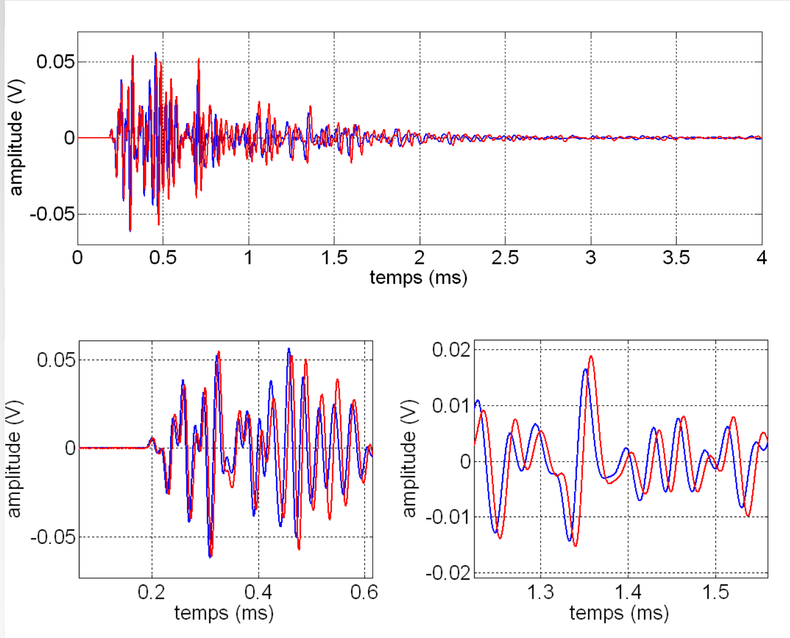




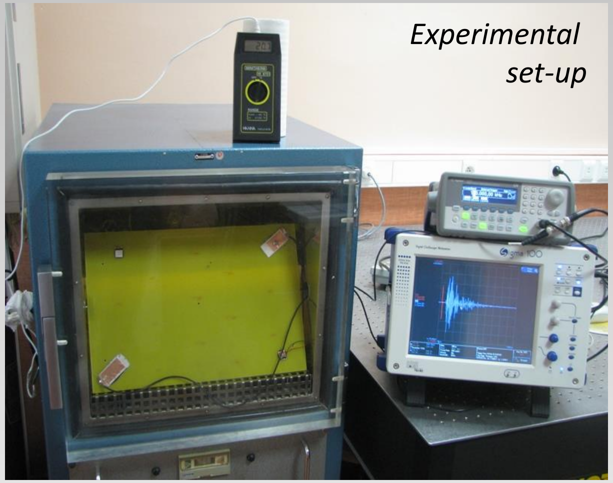
Experimental validation #2

Application to composite material

- Material: glass-epoxy FR4
- Estimator: short-time Xcorrelation
- Temperature controlled
- Embedded transducers

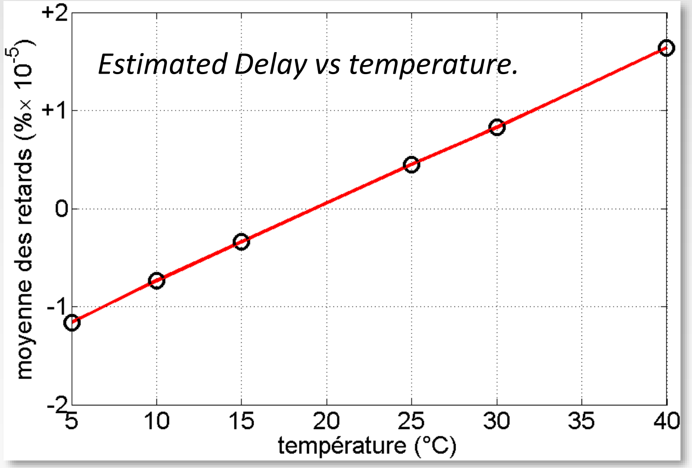


Glass-Epoxy (FR4) composite plate



Experimental set-up

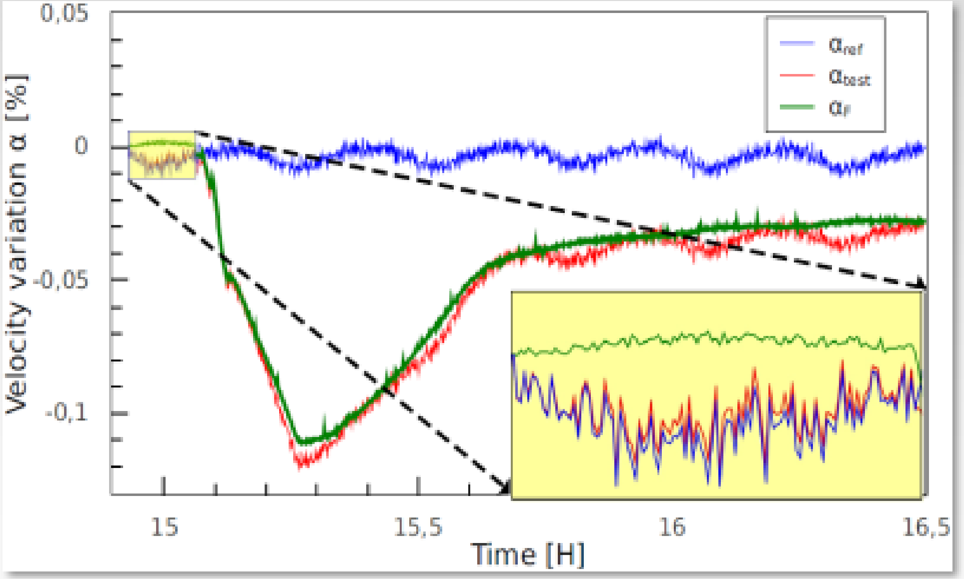
Embedded transducer

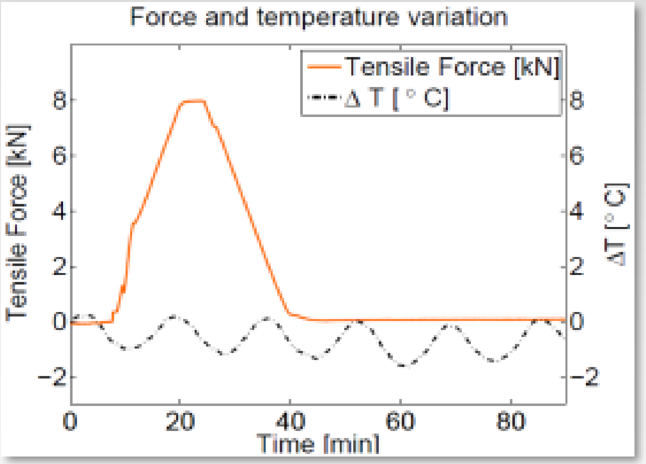


Experimental validation #3

Application to concrete

- Study of the behaviour of concrete specimens under axial loading
- Temperature compensation using a reference specimen





Conclusions

- CRBs give a good indication of the expected accuracy of a particular configuration of a SHM system as a function of
  - the geometry of a transducers array;
  - the SNR of the data acquisition.
- The approximated expression for the CRB provides a way to select optimal values for the signal parameters, especially for the sampling period